1) $\boxed{f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$

2) $f'(x) = \lim_{h \to 0} \frac{\left[2(x+h)^2 + 5(x+h) - 1\right] - \left[2x^2 + 5x - 1\right]}{h}$

$= \lim_{h \to 0} \frac{\left[\cancel{2x^2} + 4xh + 2h^2 + \cancel{5x} + 5h - \cancel{1}\right] - \left[\cancel{2x^2} + \cancel{5x} - \cancel{1}\right]}{h}$

$= \lim_{h \to 0} \frac{4xh + 2h^2 + 5h}{h} = \lim_{h \to 0} \frac{\cancel{h}(4x + 2h + 5)}{\cancel{h}}$

$= \lim_{h \to 0} 4x + \cancel{2h}^{\to 0} + 5 = \boxed{4x + 5}$

3) $f(t) = 2t^{-1/2} + 4t^{-3/2} + 2$

$f'(t) = 2\left(-\frac{1}{2} t^{-3/2}\right) + 4\left(-\frac{3}{2} t^{-5/2}\right) = -t^{-3/2} - 6t^{-5/2}$

$= \boxed{\frac{-1}{t^{3/2}} - \frac{6}{t^{5/2}}}$

4) $g(s) = 2s^2 - \frac{4}{s} + \frac{2}{\sqrt{s}} = 2s^2 - 4s^{-1} + 2s^{-1/2}$

$g'(s) = 4s + 4s^{-2} - s^{-3/2} = \boxed{4s + \frac{4}{s^2} - \frac{1}{s^{3/2}}}$

5) $h(x) = (2x^2 + x)^3$

$h'(x) = 3(2x^2 + x)^2 \underbrace{(4x + 1)}_{\text{chain}} = \boxed{(12x + 3)(2x^2 + x)^2}$

6) $g(t) = \dfrac{t}{t^2+4}$ Find $g''(t)$ ... use quotient rule.

$$g'(t) = \frac{(t^2+4)(1) - t(2t)}{(t^2+4)^2} = \frac{t^2+4-2t^2}{(t^2+4)^2}$$

$$= \frac{-t^2+4}{(t^2+4)^2}$$

$$g''(t) = \frac{(t^2+4)^2(-2t) - (-t^2+4)\left[2(t^2+4)\underbrace{(2t)}_{\text{chain}}\right]}{(t^2+4)^4}$$

$$= \frac{\cancel{(t^2+4)}(2t)\left[-1 - (-t^2+4)(2)\right]}{(t^2+4)^{\cancel{4}\,3}}$$

$$= \frac{2t\left(-1+2t^2-8\right)}{(t^2+4)^3} = \boxed{\frac{2t(2t^2-9)}{(t^2+4)^3}}$$

7) $f(x) = x(x^2+1)^3$ (PRODUCT RULE)

$$f'(x) = x'(x^2+1)^3 + x((x^2+1)^3)'$$

$$= (x^2+1)^3 + x\left(3(x^2+1)^2(2x)\right)$$

$$= (x^2+1)^3 + 6x^2(x^2+1)^2$$

$$= (x^2+1)^2\left[(x^2+1)+6x^2\right]$$

$$= (x^2+1)^2\left[7x^2+1\right]$$

$$f''(x) = \left(2(x^2+1)(2x)\right)(7x^2+1) + (x^2+1)^2(14x)$$

$$= 2x(x^2+1)\left[2(7x^2+1) + 7x(x^2+1)\right]$$

$$= \boxed{2x(x^2+1)(7x^3+14x^2+7x+2)}$$

8.) $f(t) = 46.9(1+1.09t)^{0.1}$

$t = 0$ Corresponds to 1900

$\therefore$ $t = 100$ Corresponds to 2000.

$$f(100) = 46.9\left(1+1.09(100)\right)^{0.1}$$

$$= \boxed{75.0433 \text{ years}}$$

Rate of Change: $f'(t) = 46.9\left((0.1)(1+1.09t)^{-0.99}\underbrace{(1.09)}_{\text{Chain}}\right)$

$$f'(100) = (46.9)(0.1)(1.09)\left(1+1.09(100)\right)^{-0.99}$$

$$= \boxed{.04871 \text{ years}/\text{year}}$$

9) $f(x) = \sqrt{x-1} = (x-1)^{1/2}$

$f'(x) = \frac{1}{2}(x-1)^{-1/2} = \frac{1}{2\sqrt{x-1}}$

Critical point @ $x = 1$.

$f'$: XXXXX | +

-2 1 3

Undefined

a) Increasing on $(1, \infty)$.

b) Min @ $x = 1$.

$f''(x) = -\frac{1}{4}(x-1)^{-3/2} = \frac{-1}{4(x-1)^{3/2}}$

$f''$: XXXXXX | +

undefined 1 5

c) Concave ↑ on $(1, \infty)$

d) No inflection points.

10) $f(x) = x^4 - 2x^2$

$f'(x) = 4x^3 - 4x := 0$

$\longrightarrow 4x(x^2-1) = 0$

$\longrightarrow 4x(x+1)(x-1) = 0$

$\longrightarrow$ C.P. : $x = 0, -1, 1$.

$f'$: $-6$, $-1$, $-\frac{1}{2}$, $0$, $\frac{1}{2}$, $1$, $3$

a) Increasing on $(-1,0) \cup (1,\infty)$
Decreasing on $(-\infty,-1) \cup (0,1)$

b) Max @ $x=0$
Min @ $x=-1, 1$.

$f''(x) = 12x^2 - 4 := 0$

$\longrightarrow 4(3x^2-1) = 0$

C.P.'s $\longrightarrow x = \pm\sqrt{\frac{1}{3}} = \pm\frac{\sqrt{3}}{3}$

$+ \quad - \quad +$

$-4$, $-\frac{\sqrt{3}}{3}$, $0$, $\frac{\sqrt{3}}{3}$, $5$

c) Concave $\uparrow$ on $(-\infty, -\frac{\sqrt{3}}{3}) \cup (\frac{\sqrt{3}}{3}, \infty)$
Concave $\downarrow$ on $(-\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3})$

d) I.P. @ $x = \pm\frac{\sqrt{3}}{3}$.

---

11) $f(x) = -(1+x^2)^{-1}$

$f'(x) = (1+x^2)^{-2}(2x)$

$= \dfrac{2x}{(1+x^2)^2}$

C.P. : $x = 0$

$-2$, $0$, $+1$

a) Inc @ $(0,\infty)$
Dec @ $(-\infty,0)$

b) Min @ $x=0$

$$f''(x) = \underbrace{2(1+x^2)^{-2} + 2x\left(-2(1+x^2)^{-3}\underbrace{(2x)}_{\text{chain}}\right)}_{\text{Product Rule}}$$

$$= 2(1+x^2)^{-3}\left[(1+x^2) + 2x^2(-2)\right]$$

$$= 2(1+x^2)^{-3}(-3x^2+1)$$

$$= \frac{2(1-3x^2)}{(1+x^2)^{+3}}$$

C.P.'s : $1-3x^2 = 0 \longrightarrow x = \pm\sqrt{\frac{1}{3}} = \frac{\pm\sqrt{3}}{3}$

$- \quad + \quad -$

$-2$, $-\frac{\sqrt{3}}{3}$, $0$, $\frac{\sqrt{3}}{3}$, $2$

c) C$\uparrow$ @ $(-\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3})$
C$\downarrow$ @ $(-\infty, -\frac{\sqrt{3}}{3}) \cup (\frac{\sqrt{3}}{3}, \infty)$

d) IP's @ $x = \pm\frac{\sqrt{3}}{3}$.

12) $f(x) = 2x^2 + 3x - 2$

$f'(x) = 4x + 3 \quad := 0$

$\longrightarrow \quad x = -\frac{3}{4}$

$f' \quad \overset{-}{\longleftarrow} \; \underset{-3/4}{|} \; \overset{+}{\longrightarrow}$

$-1 \qquad -\frac{3}{4} \qquad 0$

$\boxed{\therefore \text{ Absolute Min @ } x = -\tfrac{3}{4}}$

13) $f(x) = \frac{1}{3}x^3 - x^2 + x + 1$ over $[0, 2]$

$f'(x) = x^2 - 2x + 1 \quad := 0$

$\longrightarrow \quad (x-1)^2 = 0$

$\longrightarrow \quad x = 1$.

$f(0) = 1$, $f(1) = \frac{1}{3} - \cancel{1} + \cancel{1} + 1 = \frac{4}{3}$, $f(2) = \frac{1}{3}(8) - 4 + 2 + 1 = \frac{8}{3} - 1 = \frac{5}{3}$

$$\boxed{\begin{array}{l} \text{Abs Max @ } x = 2 \\ \text{Abs Min @ } x = 0 \end{array}}$$

14) $f(x) = \frac{x^2}{x-1}$ over $[-1, 3]$

$$f'(x) = \frac{(x-1)(2x) - x^2(1)}{x-1} = \frac{2x^2 - 2x - x^2}{x-1} = \frac{x^2 - 2x}{x-1} = \frac{x(x-2)}{x-1}$$

CP @ $x = 0, 1, 2$.

$\uparrow$ Thrown out. (B/c it's a vertical asymptote)

... Argh!

$$\boxed{\begin{array}{l} \text{No abs. Max} \\ \text{No abs Min} \end{array}}$$

(Just Graph it and you'll see)

15) $P(x) = -0.04x^2 + 240x - 10{,}000$

$P'(x) = -0.08x + 240 \quad ; = 0$

$\longrightarrow \quad x = \dfrac{-240}{-0.08} = 3{,}000$

$P''(x) = -0.08 < 0 \longleftarrow$ Concave Down Everywhere.

By the second derivative test,
$x = 3{,}000$ is a absolute max. (profit)

$\longrightarrow$ $\boxed{\text{Set the price to \$3,000}}$

16) $2x^2 - 3xy = 4$

$\longrightarrow \dfrac{d}{dx}\left(2x^2 - 3xy\right) = \dfrac{d}{dx}(4) \longrightarrow 4x\cancelto{1}{\dfrac{dx}{dx}} - 3\underbrace{\left(x\dfrac{dy}{dx} + \cancelto{1}{\dfrac{dx}{dx}}\cdot y\right)}_{\text{Product Rule}} = 0$

$\longrightarrow 4x - 3\left(x\dfrac{dy}{dx} + y\right) = 0$

$\longrightarrow 4x - 3x\dfrac{dy}{dx} - 3y = 0$

$\longrightarrow \boxed{\dfrac{dy}{dx} = \dfrac{3y - 4x}{-3x}}$

17) $x^2 + 2x^2y^2 + y^3 = 10 \longrightarrow \dfrac{d}{dx}\left(x^2 + 2x^2y^2 + y^3\right) = \dfrac{d}{dx}(10)$

$\longrightarrow 2x + \left(2xy^2 + 2x^2y\dfrac{dy}{dx}\right) + 3y^2\dfrac{dy}{dx} = 0$

$\longrightarrow \boxed{\dfrac{dy}{dx} = \dfrac{-2x - 2xy^2}{2x^2y + 3y^2}}$

17') $x^2 - y^2 = 12 + 4xy$

$$\frac{d}{dx}\left(x^2 - y^2\right) = \frac{d}{dx}\left(12 + 4xy\right)$$

$$\longrightarrow 2x - 2y\frac{dy}{dx} = 0 + 4x\frac{dy}{dx} + 4y$$

$$\longrightarrow 2x - 4y = 4x\frac{dy}{dx} + 2y\frac{dy}{dx}$$

$$\longrightarrow \boxed{\frac{2x-4y}{4x+2y} = \frac{dy}{dx}}$$

18) $h(x) = e^{4x^2-6x+1} + 3x - 1$

$$h'(x) = e^{4x^2-6x+1} \cdot \underbrace{(8x-6)}_{\text{Chain}} + 3$$

$$= \boxed{(8x-6)e^{4x^2-6x+1} + 3}$$

19) $g(x) = \ln\left(3x^2 - x + 2\right)$

$$g'(x) = \frac{\left(3x^2 - x + 2\right)'}{3x^2 - x + 2} = \boxed{\frac{6x-1}{3x^2-x+2}}$$

20) $y = e^{-2x}$ @ $(1, e^{-2})$

$y' = -2e^{-2x}$. At $x = 1$, $m = -2e^{-2(1)} = -2e^{-2}$.

Point Slope Form: $\boxed{y - e^{-2} = -2e^{-2}(x-1)}$

21) $f(x)=\frac{x(x^2-1)^2}{x-1} \longrightarrow y=\frac{x(x^2-1)^2}{x-1}=\frac{x(x+1)^2(x-1)^2}{x-1}$

$\longrightarrow y = x(x+1)^2(x-1)$.

$\ln y = \ln\left(x(x+1)^2(x-1)\right) = \ln x + 2\ln(x+1) + \ln(x-1)$

$\xrightarrow{\left(\frac{d}{dx}\right)} \frac{1}{y}\frac{dy}{dx} = \frac{1}{x} + \frac{2}{x+1} + \frac{1}{x-1}$

$\longrightarrow \frac{dy}{dx} = y\left(\frac{1}{x}+\frac{2}{x+1}+\frac{1}{x-1}\right) = x(x+1)^2(x-1)\left(\frac{1}{x}+\frac{2}{x+1}+\frac{1}{x-1}\right)$

$= \boxed{(x+1)^2(x-1) + 2x(x+1)(x-1) + x(x+1)^2}$

22) $g(t)=\frac{\ln t}{t} \longrightarrow g'(t)=\frac{t\left(\frac{1}{t}\right)-\ln t}{t^2}=\frac{1-\ln t}{t^2}$.

Critical points: $t=0, e$.

thrown out; not inside [1,2].

$$\boxed{\begin{array}{ll} g(1)=\frac{\ln 1}{1}=0 & \longleftarrow \text{Min} \\ g(2)=\frac{\ln 2}{2} & \longleftarrow \text{Max.} \end{array}}$$

23) $Q(t) = \dfrac{3000}{1+499e^{-kt}}$. We have $Q(10) = 90$.

$$\longrightarrow 90 = \frac{3000}{1+499e^{-k(10)}} \longrightarrow 1+499e^{-10k} = \frac{3000}{90}$$

$$\longrightarrow 499e^{-10k} = \frac{3000}{90} - 1$$

$$\longrightarrow e^{-10k} = \frac{\left(\frac{3000}{90} - 1\right)}{499}$$

$$\longrightarrow -10k = \ln\left(\frac{\frac{3000}{90} - 1}{499}\right) \longrightarrow k = \frac{-1}{10}\ln\left(\frac{\frac{3000}{90} - 1}{499}\right)$$

$$\boxed{k \approx .274}$$

$$\therefore Q(t) = \frac{3000}{1+499e^{-.274t}}$$

$$@\ t = 20 : \quad Q(20) = \frac{3000}{1+499e^{-.274(20)}} \boxed{\approx 969.281}$$

$\boxed{\text{Approximately 969 students.}}$

24) $\displaystyle\int (x^3 + 2x^2 - x)\,dx = \boxed{\tfrac{1}{4}x^4 + \tfrac{2}{3}x^3 - \tfrac{1}{2}x^2 + C}$

25) $\displaystyle\int (3x-1)(3x^2-2x+1)\,dx$

Let $u = 3x^2 - 2x + 1$ ; $du = (6x-2)\,dx \longrightarrow \frac{1}{2}du = (3x-1)\,dx$

$$= \frac{1}{2}\int u\,du = \frac{1}{2}\frac{1}{2}u^2 + C = \boxed{\frac{1}{4}\left(3x^2-2x+1\right)^2 + C}$$

26) $\int \left(x^4 - 2x^3 + \frac{1}{x^2}\right) dx = \int (x^4 - 2x^3 + x^{-2})\, dx$

$= \frac{1}{5}x^5 - \frac{1}{2}x^4 - x^{-1} + C$

$= \boxed{\frac{1}{5}x^5 - \frac{1}{2}x^4 - \frac{1}{x} + C}$

27) $\int (x^4 + 2x^3 - \frac{1}{x})\, dx = \boxed{\frac{1}{5}x^5 + \frac{1}{2}x^4 - \ln|x| + C}$

28.) $\int \sqrt{2x+1}\, dx$

Let $u = 2x+1$ ; $\frac{1}{2}du = dx$

$= \frac{1}{2}\int \sqrt{u}\, du = \frac{1}{2}\left(\frac{2}{3}u^{3/2}\right) + C$

$= \boxed{\frac{1}{3}(2x+1)^{3/2} + C}$

29.) $\int \frac{x}{\sqrt{x-2}}\, dx$

Let $u = x-2$ ; $du = dx$

Then: $u+2 = x$

$= \int \frac{u+2}{\sqrt{u}}\, du = \int \left(\sqrt{u} + 2u^{-1/2}\right) du$

$= \frac{2}{3}u^{3/2} + 4u^{1/2} + C$

$= \boxed{\frac{2}{3}(x-2)^{3/2} + 4(x-2)^{1/2} + C}$

30) $\int (x+\frac{1}{2}) e^{x^2+x+1} \, dx$

Let $u = x^2+x+1$ ; $du = (2x+1)dx \longrightarrow \frac{1}{2}du = (x+\frac{1}{2})dx$.

$\frac{1}{2}\int e^u du = \frac{1}{2}e^u + C = \boxed{\frac{1}{2}\left(e^{x^2+x+1}\right) + C}$

31) $\int_0^1 (2x^3+1)\,dx = \frac{1}{2}x^4 + x \Big|_0^1 = \left[\frac{1}{2}(1)^4 + 1\right] - \left[\frac{1}{2}(0)^4 + 0\right]$

$= \boxed{3/2}$

32) $\int_{-1}^{0} \frac{e^{-x}}{(1+e^{-x})^2} dx$ ; Let $u = 1+e^{-x}$ ; $du = -e^{-x}dx$

$\longrightarrow -du = e^{-x}dx$

$x = -1 \longrightarrow u = 1+e$

$x = 0 \longrightarrow u = 1+1 = 2$

$= -\int_{1+e}^{2} \frac{du}{u^2} = \int_{2}^{1+e} \frac{du}{u^2} = \int_2^{1+e} u^{-2} du$

$= -u^{-1}\Big|_2^{1+e}$

$= -\left(\frac{1}{1+e} - \frac{1}{2}\right)$

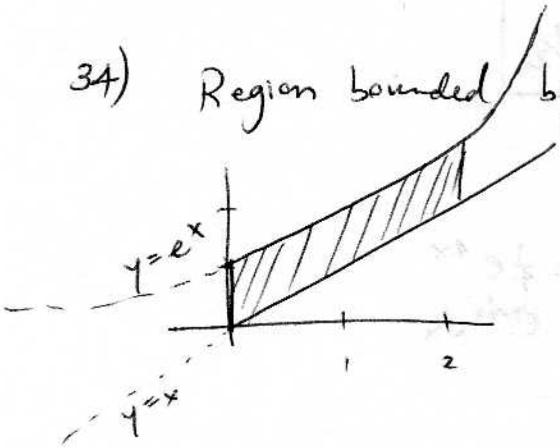
$= \boxed{\frac{1}{2} - \frac{1}{1+e}}$

33.) $\int_1^e \frac{\ln x}{x}\,dx$  let $u = \ln x$ ; $du = \frac{1}{x}\,dx$

$x = 1 \longrightarrow u = 0$
$x = e \longrightarrow u = 1$

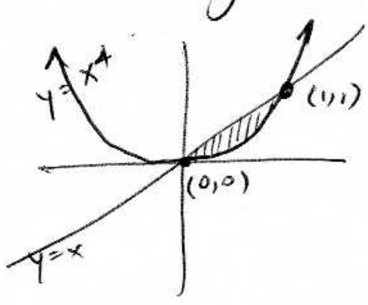
$$= \int_0^1 u\,du = \frac{1}{2}u^2 \Big|_0^1 = \boxed{\frac{1}{2}}$$

34) Region bounded by $f(x) = e^x$, $g(x) = x$, $x = 0$, $x = 2$.



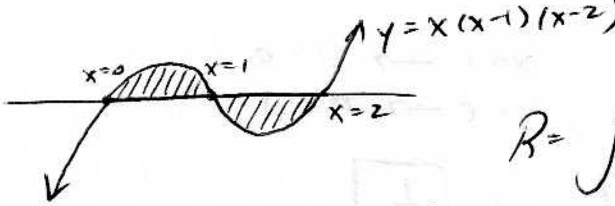

$$R = \int_0^2 (\text{top} - \text{bottom})\,dx$$
$$= \int_0^2 (e^x - x)\,dx$$
$$= e^x - \frac{1}{2}x^2 \Big|_0^2$$
$$= [e^2 - 2] - [1 - 0]$$
$$= \boxed{e^2 - 3}$$

35.) Region enclosed by $f(x) = x^4$, $g(x) = x$.




$$R = \int_0^1 (x - x^4)\,dx = \frac{1}{2}x^2 - \frac{1}{5}x^5 \Big|_0^1$$
$$= \frac{1}{2} - \frac{1}{5} = \boxed{\frac{3}{10}}$$

36) Region bounded by $y=x(x-1)(x-2)$ & $y=0$.




$$R = \int_0^1 \left[x(x-1)(x-2) - 0\right]dx + \int_1^2 \left[0 - x(x-1)(x-2)\right]dx$$

$$= \int_0^1 (x^3 - 3x^2 + 2x)\,dx + \int_1^2 (-x^3 + 3x^2 - 2x)\,dx$$

$= \text{Calculator} = \boxed{\tfrac{1}{2}}$.

37) $\int x e^{4x}\,dx$. Let $u = x$, $du = dx$; $v = \frac{1}{4}e^{4x}$, $dv = e^{4x}\,dx$

$uv - \int v\,du \ldots$

$$= \frac{1}{4}xe^{4x} - \int e^{4x}\,dx$$

$$= \boxed{\frac{1}{4}xe^{4x} - \frac{1}{4}e^{4x} + C}$$

Constant!

38) $\int \ln(2x)\,dx = \int (\ln 2 + \ln x)\,dx = \int \ln 2\,dx + \underbrace{\int \ln x\,dx}_{\text{parts}}$

$\int \ln x\,dx$ Let $u = \ln x$, $du = \frac{1}{x}dx$; $v = x$, $dv = dx$

$$= x\ln x - \int dx$$

$$= x\ln x - x.$$

Original $= \int \ln 2\,dx + \int \ln x\,dx$

$$= \boxed{\ln 2\, x + (x\ln x - x) + C}$$

39) $\int x^2 e^{2x}\,dx$

Let $u_1 = x^2 \qquad v_1 = \frac{1}{2}e^{2x}$

$du_1 = 2x\,dx \qquad dv_1 = e^{2x}\,dx$

(Parts)

$$= \frac{1}{2}x^2e^{2x} - \frac{1}{2}\int e^{2x}(2x\,dx)$$

$$= \frac{1}{2}x^2e^{2x} - \underbrace{\int xe^{2x}\,dx}_{\text{parts again.}}$$

Let $u_2 = x \qquad v_2 = \frac{1}{2}e^{2x}$

$du_2 = dx \qquad dv_2 = e^{2x}\,dx$

$$= \frac{1}{2}x^2e^{2x} - \left(\frac{1}{2}xe^{2x} - \frac{1}{2}\int e^{2x}\,dx\right)$$

$$= \frac{1}{2}x^2e^{2x} - \left(\frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x}\right) + C$$

$$= \frac{1}{2}x^2e^{2x} - \frac{1}{2}xe^{2x} + \frac{1}{4}e^{2x} + C$$

$$= \boxed{\frac{1}{4}e^{2x}\left(2x^2 - 2x + 1\right) + C}$$

40.) $\int_1^\infty e^{-2x}\,dx = \lim_{b\to\infty}\int_1^b e^{-2x}\,dx$

$$= \lim_{b\to\infty}\left(-\frac{1}{2}e^{-2x}\Big|_1^b\right)$$

$$= \lim_{b\to\infty}\left(\frac{-1}{2e^{2x}}\Big|_1^b\right) = \lim_{b\to\infty}\left(\cancel{\frac{-1}{2e^{2b}}}^{\,0} + \frac{1}{2e^2}\right)$$

$$= \boxed{\frac{1}{2e^2}}$$

41) $\int_{-\infty}^{\infty} x e^{1-x^2}\,dx = \lim_{a\to\infty} \int_{-a}^{a} x e^{1-x^2}\,dx$

Let $u = 1-x^2$

$du = -2x\,dx \longrightarrow -\frac{1}{2}du = x\,dx$.

$a \longrightarrow 1-a^2$

$-a \longrightarrow 1-(-a)^2 = 1-a^2$.

$\xrightarrow{\text{Substitute}} \lim_{a\to\infty} \int_{1-a^2}^{1-a^2} -\frac{1}{2}e^{u}\,du$

(Notice upper and lower limits of integration are the same!)

$= \boxed{0}$

42) $\begin{cases} x+y = 12 \\ x+3y = 18 \end{cases} \longrightarrow \left(\begin{array}{cc|c} 1 & 1 & 12 \\ 1 & 3 & 18 \end{array}\right) \xrightarrow{R_2 = r_2 - r_1} \left(\begin{array}{cc|c} 1 & 1 & 12 \\ 0 & 2 & 6 \end{array}\right)$

$\xrightarrow{R_2 = \frac{1}{2}r_2} \left(\begin{array}{cc|c} 1 & 1 & 12 \\ 0 & 1 & 3 \end{array}\right) \xrightarrow{R_1 = r_1 - r_2} \left(\begin{array}{cc|c} 1 & 0 & 9 \\ 0 & 1 & 3 \end{array}\right)$

$\longrightarrow$ $\boxed{\text{Solution}: (9, 3)}$

43) $\begin{cases} x + y = 12 \\ 2x - y = 18 \end{cases} \longrightarrow \left(\begin{array}{cc|c} 1 & 1 & 12 \\ 2 & -1 & 18 \end{array}\right) \xrightarrow{R_2 = r_2 - 2r_1} \left(\begin{array}{cc|c} 1 & 1 & 12 \\ 0 & -3 & -6 \end{array}\right)$

$\xrightarrow{R_2 = -\frac{1}{3} r_2} \left(\begin{array}{cc|c} 1 & 1 & 12 \\ 0 & 1 & 2 \end{array}\right) \xrightarrow{R_1 = r_1 - r_2} \left(\begin{array}{cc|c} 1 & 0 & 10 \\ 0 & 1 & 2 \end{array}\right)$

$\longrightarrow$ $\boxed{\text{Solution : } (10, 2)}$

44) $\begin{cases} x_1 - 2x_3 = 0 \\ -x_1 + x_2 + 2x_3 = 1 \\ -x_1 + 2x_2 + 3x_3 + x_4 = 7 \\ x_1 - 2x_3 + x_4 = 4 \end{cases} \longrightarrow \left(\begin{array}{cccc|c} 1 & 0 & -3 & 0 & 0 \\ -1 & 1 & 2 & 0 & 1 \\ -1 & 2 & 3 & 1 & 7 \\ 1 & 0 & -2 & 1 & 4 \end{array}\right)$

$\xrightarrow[R_3 = r_3 + r_1]{R_2 = r_2 + r_1} \left(\begin{array}{cccc|c} 1 & 0 & -3 & 0 & 0 \\ 0 & 1 & -1 & 0 & 1 \\ 0 & 2 & 0 & 1 & 7 \\ 1 & 0 & -2 & 1 & 4 \end{array}\right) \xrightarrow{R_4 = r_4 - r_1} \left(\begin{array}{cccc|c} 1 & 0 & -3 & 0 & 0 \\ 0 & 1 & -1 & 0 & 1 \\ 0 & 2 & 0 & 1 & 7 \\ 0 & 0 & 1 & 1 & 4 \end{array}\right)$

$\xrightarrow{R_3 = r_3 - 2r_2} \left(\begin{array}{cccc|c} 1 & 0 & -3 & 0 & 0 \\ 0 & 1 & -1 & 0 & 1 \\ 0 & 0 & 2 & 1 & 5 \\ 0 & 0 & 1 & 1 & 4 \end{array}\right) \xrightarrow{R_3 \leftrightarrow R_4} \left(\begin{array}{cccc|c} 1 & 0 & -3 & 0 & 0 \\ 0 & 1 & -1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 4 \\ 0 & 0 & 2 & 1 & 5 \end{array}\right)$

$\xrightarrow[R_2 = r_2 + r_3,\ R_4 = r_4 - 2r_3]{R_1 = r_1 + 3r_3} \left(\begin{array}{cccc|c} 1 & 0 & 0 & 3 & 12 \\ 0 & 1 & 0 & 1 & 5 \\ 0 & 0 & 1 & 1 & 4 \\ 0 & 0 & 0 & -1 & -3 \end{array}\right) \xrightarrow{R_4 = -r_4} \left(\begin{array}{cccc|c} 1 & 0 & 0 & 3 & 12 \\ 0 & 1 & 0 & 1 & 5 \\ 0 & 0 & 1 & 1 & 4 \\ 0 & 0 & 0 & 1 & 3 \end{array}\right)$

$\xrightarrow[R_2 = r_2 - r_4,\ R_3 = r_3 - r_4]{R_1 = r_1 - 3r_4} \left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 3 \end{array}\right) \longrightarrow$ $\boxed{\text{Solution : } (3, 2, 1, 3)}$

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 0 \\ 0 & -1 \\ 1 & 0 \\ 0 & -1 \end{pmatrix}$$

45) $$AB = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \\ 1 & 0 \\ 0 & -1 \end{pmatrix} = \boxed{\begin{pmatrix} 4 & -6 \\ 6 & -4 \end{pmatrix}}$$

46) $$A^T + B = \begin{pmatrix} 1 & 4 \\ 2 & 3 \\ 3 & 2 \\ 4 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & -1 \\ 1 & 0 \\ 0 & -1 \end{pmatrix} = \boxed{\begin{pmatrix} 2 & 4 \\ 2 & 2 \\ 4 & 2 \\ 4 & 0 \end{pmatrix}}$$

47) $$(BA)^T = \left[\begin{pmatrix} 1 & 0 \\ 0 & -1 \\ 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \end{pmatrix}\right]^T$$

$$= \begin{pmatrix} 1 & 2 & 3 & 4 \\ -4 & -3 & -2 & -1 \\ 1 & 2 & 3 & 4 \\ -4 & -3 & -2 & -1 \end{pmatrix}^T = \boxed{\begin{pmatrix} 1 & -4 & 1 & -4 \\ 2 & -3 & 2 & -3 \\ 3 & -2 & 3 & -2 \\ 4 & -1 & 4 & -1 \end{pmatrix}}$$

48) $$(A + B^T)^T = A^T + B^{TT} = \underbrace{A^T + B}_{\text{This is } \#46.} = \boxed{\begin{pmatrix} 2 & 4 \\ 2 & 2 \\ 4 & 2 \\ 4 & 0 \end{pmatrix}}$$

49) $$\begin{pmatrix} 1 & 2 & 3 \\ 1 & 1 & 1 \\ 0 & 1 & 3 \end{pmatrix}\begin{pmatrix} -2 & 3 & 1 \\ 3 & -3 & -2 \\ -1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} -2 & 3 & 1 \\ 3 & -3 & -2 \\ -1 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 & 3 \\ 1 & 1 & 1 \\ 0 & 1 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$\boxed{\text{Yes.}}$

50.) $$\begin{pmatrix} -3 & 0 & 1 \\ 1 & 3 & 1 \\ -3 & 2 & 2 \end{pmatrix}\begin{pmatrix} -4 & -2 & 3 \\ 5 & 3 & -4 \\ -11 & -6 & 8 \end{pmatrix} = \begin{pmatrix} 1 & 0 & -1 \\ & & \\ & & \end{pmatrix}$$

$\boxed{\text{No!}}$

51.) $$\begin{cases} 13x_1 + 4x_2 = 33 \\ 3x_1 + x_2 = 8 \end{cases}$$

$$A = \begin{pmatrix} 13 & 4 \\ 3 & 1 \end{pmatrix} ; X = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} ; B = \begin{pmatrix} 33 \\ 8 \end{pmatrix}$$

Finding $A^{-1}$ : $$\left(\begin{array}{cc|cc} 13 & 4 & 1 & 0 \\ 3 & 1 & 0 & 1 \end{array}\right) \xrightarrow[R_1 = r_1 - 4r_2]{} \left(\begin{array}{cc|cc} 1 & 0 & 1 & -4 \\ 3 & 1 & 0 & 1 \end{array}\right)$$

$$\xrightarrow[R_2 = r_2 - 3r_1]{} \left(\begin{array}{cc|cc} 1 & 0 & 1 & -4 \\ 0 & 1 & -3 & 13 \end{array}\right)$$

$$\longrightarrow A^{-1} = \begin{pmatrix} 1 & -4 \\ -3 & 13 \end{pmatrix}.$$

$$AX = B \longrightarrow \underbrace{A^{-1}A}_{I}X = A^{-1}B \longrightarrow IX = A^{-1}B$$

$$\longrightarrow \boxed{X = A^{-1}B}$$

$$\longrightarrow X = \begin{pmatrix} 1 & -4 \\ -3 & 13 \end{pmatrix}\begin{pmatrix} 33 \\ 8 \end{pmatrix} = \begin{pmatrix} 1 \\ 5 \end{pmatrix}$$

$\boxed{\text{Solution } x_1 = 1, \; x_2 = 5}$