

## Math 125-126

### Supplements

In many elementary classrooms, **mathematics** is considered a synonym for **arithmetic**. This occurs despite the fact that many, if not most, of the interesting problems which can be posed to challenge young children relate to measurement, and thus indirectly to geometry. We believe that the subject of mathematics should include substantial portions of both arithmetic and geometry from the very beginning of the school experience.

Since teachers are inclined to teach material with the same emphasis they have experienced in their subject matter classes, the mathematics content in Math 125 should accordingly present arithmetic and measurement as interlinked topics, persisting throughout the course. As our text does not order the material in this way, we have created Supplements A and B to provide coverage of measurement topics paralleling, and sometimes inspiring, the arithmetic topics covered in the first several chapters in the text. Supplement A introduces measurement of line segments, angles, and areas as an application of whole number counting, while Supplement B deals with the connections between the arithmetic operations on whole numbers, and more advanced measuring techniques. Geometric ideas are introduced and used throughout these Supplements, not in the axiomatic setting often studied in high school, but in an experimental hands on setting in which the student is lead to discover properties which can later be formalized as axioms and theorems.

Supplement C, which is intended for use in Math 126, addresses the problem of formalizing the geometry which has been discovered previously, much as high school geometry builds (or should build) on the elementary school geometry experiences of the students. It can be viewed as an introduction to what might be called the scientific method in geometry, viz. observed phenomena (measure), formulate hypothesis (axioms and theorems), and test hypothesis. The emphasis is on identifying the most fundamental geometric principles (axioms) and showing how other geometric properties relevant to measurement can be deduced from these. A short section on similar triangles brings together much of the preceding material in developing a powerful tool for indirect measure of distances.

### Labs

There is convincing evidence that many students learn material best when they have the opportunity to deal with related hands on activities which present them with the opportunity to experiment, conjecture and test their conjectures while working in small groups with fellow students. The lab activities included in this packet are designed to stimulate thought about the mathematical concepts introduced in Math 125/126, and to introduce students to some “manipulatives” which are available for use in their elementary classroom period. The activities are intended for use either in full hour lab sessions accompanying a lecture course or as supplemental classroom activities for classes taught in a small class setting. They will be most effective if students work in groups, discussing among themselves the ideas raised by the activities. The goal of these labs is to involve students in the doing of mathematics. The goal is not to fill in lab sheets with “correct” answers as quickly as possible. If a particular activity is completed before the end of the period, students should look for interesting extensions which will further their learning. If the activity can not be finished during one period, it will provide a thought provoking project when completed outside of class.

Note: No labs requiring use of computers are included, though the use of such labs is desirable if sufficient computer access is available. In particular, geometry software such as Geometer's Sketchpad is very useful in motivating students to experiment and conjecture about the principles of geometry. Such software usually comes packaged with very good suggestions for its use in a laboratory setting.

These supplements have been written and revised over the years by Joe Ferrar, Joan Leitzel and Jim Schultz, with ample input from colleges. The current version has been prepared by Joe Ferrar, July, 1995 and continuously modified by Cary Lee.

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