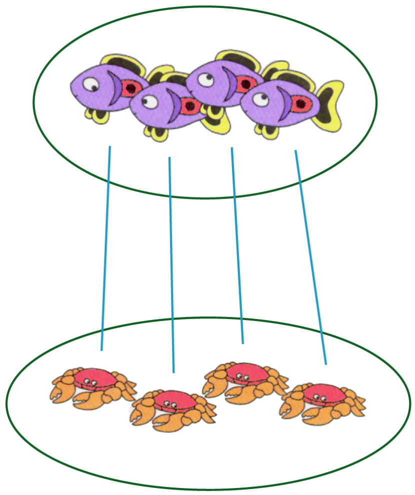
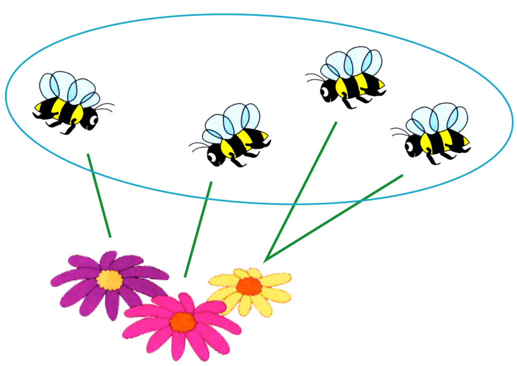
One-to-One Correspondences

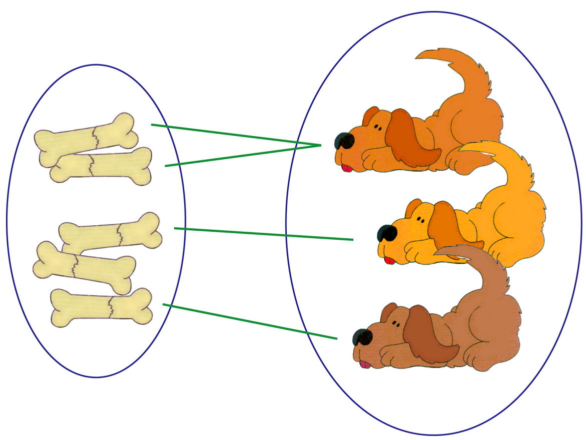
A one-to-one correspondence between two sets A and B is a pairing of objects from A and B such that every object in set A is paired up with exactly one object in B and vice versa.

The following diagram is an example of a one-to-one correspondence. It is not hard to see that we can construct many other different one-to-one correspondences between these two sets



The followings are not one-to-one correspondences because more than one object in set A are paired up with one object in set B, or some objects in set A are left out.





We know that two sets A and B have the same number of elements if there is a one-to-one correspondence between them, and in this case, we say that A and B are in one-to-one correspondence with each other, or that A and B are **equivalent**. Using this terminology, we can define the concept of a number.

The number *one* is the attribute common to all sets that are in one-to-one correspondence with the set { 🕯 }. (You can replace the candle by any object you like.)

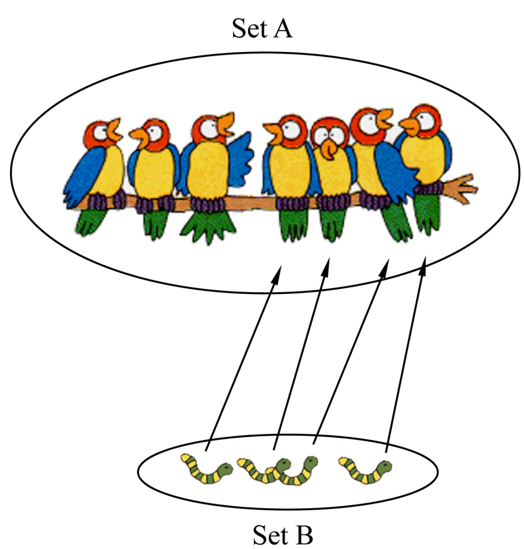
The number *two* is the attribute common to all sets that are in one-to-one correspondence with the set { 🕯, 🕯 }.

The number *three* is the attribute common to all sets that are in one-to-one correspondence with the set { 🕯, 🕯, 🕯}.

We can similarly define the concept of four, or five, or any other positive whole number.

Ordering

We say that a set *A* has more elements than another set *B* if there is a one-to-one correspondence between B and a *proper subset* of A. In other words, there is a one-to-one correspondence between B and a portion of *A* such that some elements in *A* are left unpaired by this correspondence. The following diagram illustrates this situation.



We can finally define the ordering of numbers as follow,
A number $\boldsymbol{x}$ is greater than another number $\boldsymbol{y}$ if there is a one-to-one correspondence from a set *B* with $\boldsymbol{y}$ elements into a portion of a set *A* with $\boldsymbol{x}$ elements such that there is at least one element in *A* left unpaired.