* 1. **Determinants and Cramer’s Rule**

If a matrix is square (n × n) then it has a **determinant**. Determinants are used to:

1. Determine if a matrix has an inverse matrix
2. Solve systems of linear equations (Cramer’s Rule)

**Determinant of a 2 × 2 matrix:**

Ifthen 

**Determinant of a n × n matrix:**

If **Error! Objects cannot be created from editing field codes.**,then 

where  is the cofactor of , and calculated using the formula 

where  is the minor of , and is equal to the determinant of the matrix obtained by deleting the ith row and jth column of matrix A

Example:

 Find det(A).

  









**Cramer’s Rule**

If a system of n linear equations in the n variables  is equivalent to the matrix equation *DX = B*, and if  , then its solutions are



where  is the matrix obtained by replacing the ith column of *D* by the n × 1 matrix *B*.

**Example #1:**

 converts into the matrix equation: 

By Cramer’s Rule the solutions for x and y are as follows:

 since the 1st column,is replaced by the column 

 since the 2nd column,is replaced by the column 

 and 

So, the solution to the system is (3/2, 2).

**Example #2:**

 converts into the matrix equation: 

By Cramer’s Rule the solutions for x , y and z are:

 , , 

So, the solution to the system is (4.4, 0.96, –2.64).

**Example #3: (YOU CAN USE YOUR CALCULATOR TO FIND THE DETERMINANTS)**

 converts into the matrix equation: 

By Cramer’s Rule the solutions for x , y, z and w are:

* 1. **Partial Fractions**

**Procedure for Decomposing a Rational Expression into Partial Fractions**

Consider , such that  and  have no common factors.

1. Factor  into linear factors and/or irreducible quadratic factors.

Linear factors may be distinct: 

or

Linear factors may be repeated *k* times: 

and similarly,

Quadratic factors may be distinct: 

or

Quadratic factors may be repeated *k* times: 

1. Assign to each factor a sum of partial fractions as follows:

* For each distinct linear factor , assign the partial fraction  .
* For each repeated linear factor , assign the sum of *k* partial fractions

 .

* For each distinct quadratic factor , assign the partial fraction  .
* For each repeated quadratic factor , assign the sum of k partial fractions



1. Applying algebraic methods, create a system of equations involving coefficients A, B, C, etc.
2. Solve the system for A, B, C, etc. using one of the methods from Chapter 8.
3. Using these constants in the numerators, rewrite the rational expression as a sum of partial fractions.

**Example #1: Distinct linear factors.**

Perform partial fraction decomposition on 

**Example #2: Repeated linear factors.**

Perform partial fraction decomposition on 

**Example #3: Distinct quadratic factors.**

Perform partial fraction decomposition on 

**Example #4: Repeated quadratic factors.**

Perform partial fraction decomposition on 