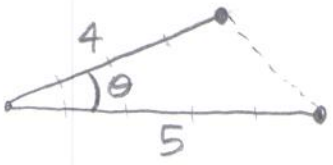
Math 180: Related Rates, 3.9 – Day 2

1. Two sides of a triangle are 4 m and 5 m in length and the angle between them is increasing at a rate of 0.06 rad/s. Find the rate at which the area of the triangle is increasing when the angle between the sides of fixed length is $\pi/3$.

Find $\frac{dA}{dt}$



$A = \frac{1}{2}ab\sin\theta$

$A = \frac{1}{2}(4)(5)\sin\theta$

$A = 10\sin\theta$

$\frac{d}{dt}[A] = \frac{d}{dt}[10\sin\theta]$

$\frac{dA}{dt} = 10\cos\theta \cdot \frac{d\theta}{dt}$

moment in time

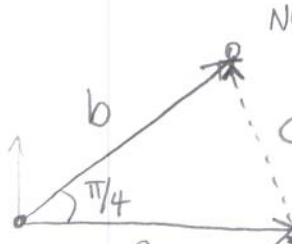
$\theta = \pi/3$ radians

$\frac{d\theta}{dt} = .06$ rad/s

Plug in: $\frac{dA}{dt} = 10\cos(\frac{\pi}{3})\cdot(.06) = \boxed{.3\ m^2/s}$

2. Two people start from the same point. One walks east at 3 mi/h and the other walks northeast at 2 mi/h. How fast is the distance between the people changing after 15 minutes?

Find $\frac{dc}{dt}$.



Law of cosines

$c^2 = a^2 + b^2 - 2ab\cos(\frac{\pi}{4})$

$c^2 = a^2 + b^2 - 2ab(\frac{\sqrt{2}}{2})$

$c^2 = a^2 + b^2 - \sqrt{2}ab$

know:

$\frac{da}{dt} = 3$ mph

$\frac{db}{dt} = 2$ mph

$\frac{d}{dt}[c^2] = \frac{d}{dt}[a^2 + b^2 - \sqrt{2}ab]$ (product rule!)

$2c\cdot\frac{dc}{dt} = 2a\frac{da}{dt} + 2b\frac{db}{dt} - \sqrt{2}(\frac{da}{dt}\cdot b + a\cdot\frac{db}{dt})$

moment in time

$a = 3(\frac{1}{4}) = 3/4$

$b = 2(1/4) = 1/2$

$c = \sqrt{(\frac{3}{4})^2 + (\frac{1}{2})^2 - \sqrt{2}(\frac{3}{4})(\frac{1}{2})}$

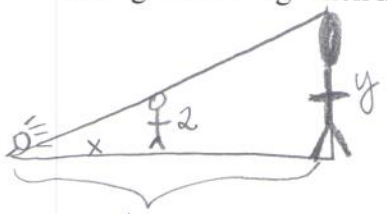
$c \approx .5312$

Plug in: $2(.5312)\frac{dc}{dt} = 2(\frac{3}{4})(3) + 2(\frac{1}{2})(2) - \sqrt{2}(3\cdot\frac{1}{2} + \frac{3}{4}(2))$

$\frac{dc}{dt} \approx \boxed{2.125\ \text{mph}}$

3. A spotlight on the ground shines on the wall of a building 12 m away. If a man 2 m tall walks from the spotlight toward the building at a speed of 1.6 m/s, how fast is the length of his shadow on the building decreasing when he is 4 m from the wall?

Find $\frac{dy}{dt}$.



By similar triangles: $\frac{12}{x} = \frac{y}{2}$

$xy = 24$

$\frac{d}{dt}[x\cdot y] = \frac{d}{dt}[24]$

$\frac{dx}{dt}\cdot y + x\cdot\frac{dy}{dt} = 0$

moment in time

$x = 12 - 4 = 8$ m

$y = \frac{24}{8} = 3$ m

$\frac{dx}{dt} = 1.6$ m/sec

Plug in: $(1.6)(3) + 8(\frac{dy}{dt}) = 0$

So $\frac{dy}{dt} = \boxed{-.6\ \text{m/s}}$