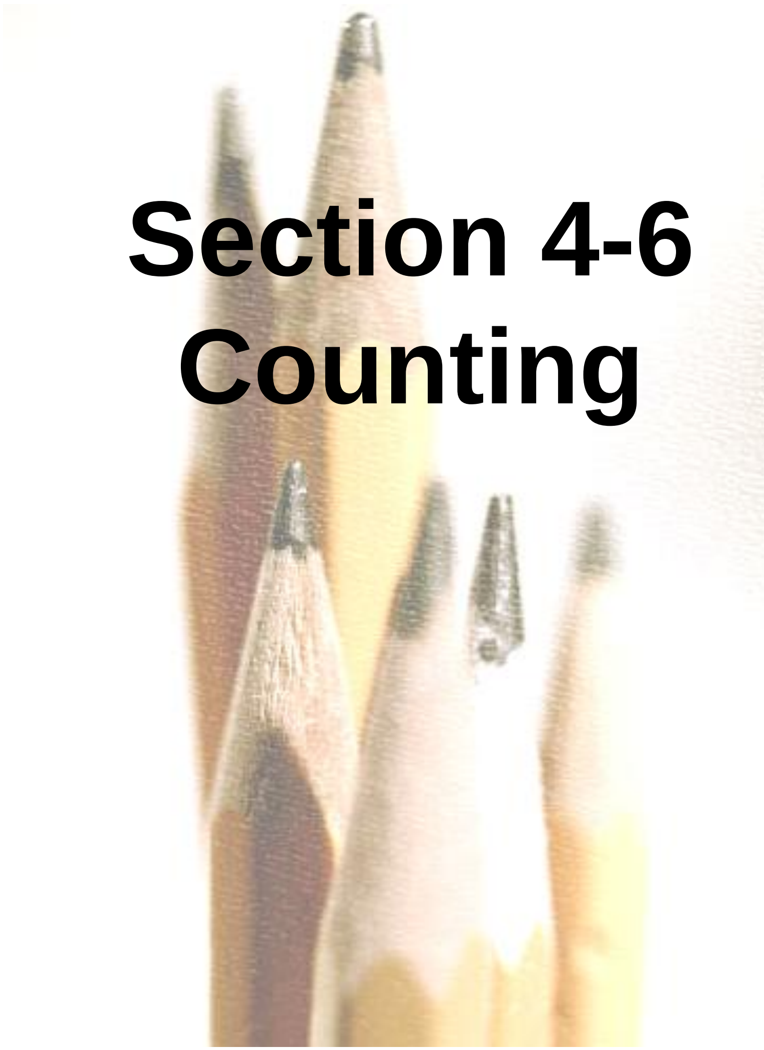


Section 4-6
Counting

Fundamental Counting Rule

For a sequence of two events in which the first event can occur m ways and the second event can occur n ways, the events together can occur a total of $m \cdot n$ ways.

Notation

The factorial symbol ! denotes the product of decreasing positive whole numbers.

For example,

$$4! = 4 \bullet 3 \bullet 2 \bullet 1 = 24.$$

By special definition, 0! = 1.

Factorial Rule

A collection of *n* different items can be arranged in order *n*! different ways. (This factorial rule reflects the fact that the first item may be selected in *n* different ways, the second item may be selected in *n* – 1 ways, and so on.)

Permutations Rule
(when items are all different)

Requirements:

1. **There are *n* different items available. (This rule does not apply if some of the items are identical to others.)**
2. **We select *r* of the *n* items (without replacement).**
3. **We consider rearrangements of the same items to be different sequences. (The permutation of ABC is different from CBA and is counted separately.)**

If the preceding requirements are satisfied, the number of permutations (or sequences) of *r* items selected from *n* available items (without replacement) is

$$_{n}P_{r} = \frac{n!}{(n-r)!}$$

Permutations Rule
(when some items are identical to others)

Requirements:

1. There are n items available, and some items are identical to others.
2. We select all of the n items (without replacement).
3. We consider rearrangements of distinct items to be different sequences.

If the preceding requirements are satisfied, and if there are n_1 alike, n_2 alike, . . . n_k alike, the number of permutations (or sequences) of all items selected without replacement is

$$\frac{n!}{n_1! \,.\, n_2! \,\ldots\ldots\ldots\, n_k!}$$

Combinations Rule

Requirements:

1. **There are n different items available.**
2. **We select r of the n items (without replacement).**
3. **We consider rearrangements of the same items to be the same. (The combination of ABC is the same as CBA.)**

If the preceding requirements are satisfied, the number of combinations of r items selected from n different items is

$$_{n}C_{r} = \frac{n!}{(n - r)!\, r!}$$

Permutations versus Combinations

When different orderings of the same items are to be counted separately, we have a permutation problem, but when different orderings are not to be counted separately, we have a combination problem.

Recap

In this section we have discussed:

- **The fundamental counting rule.**
- **The factorial rule.**
- **The permutations rule (when items are all different).**
- **The permutations rule (when some items are identical to others).**
- **The combinations rule.**

Examples

Pgs 183-185: 2, 6, 8, 14, 21, 27, 32

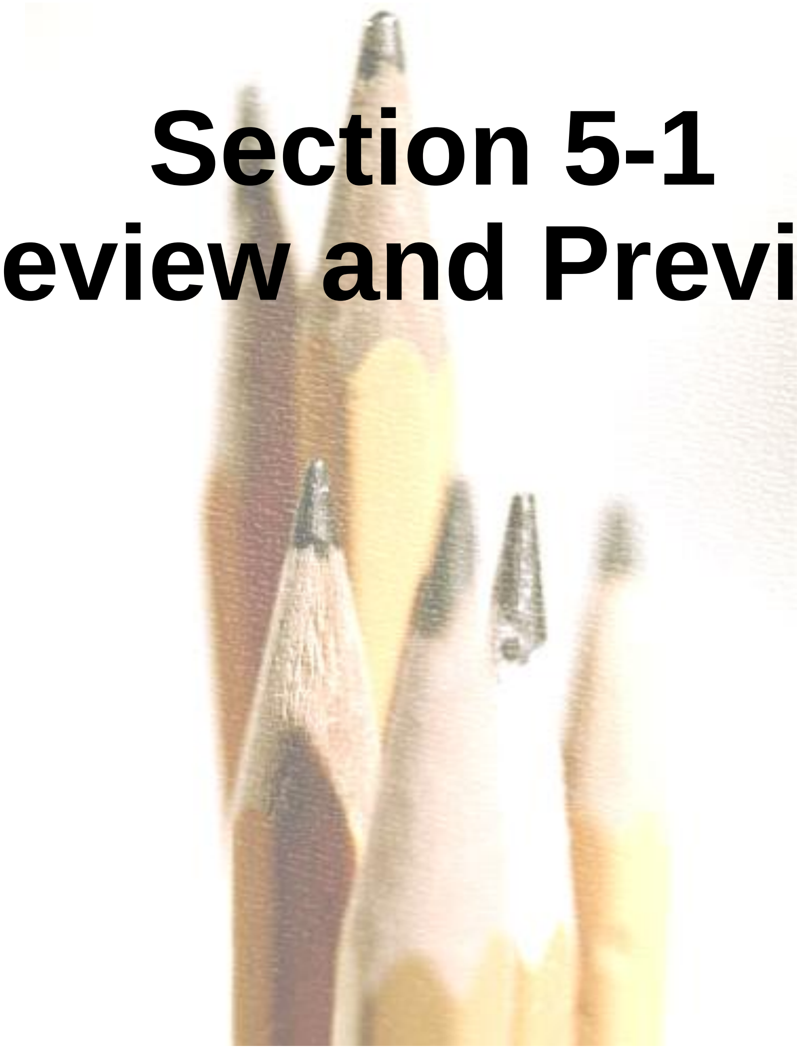
Chapter 5
Probability Distributions

5-1 Review and Preview

5-2 Random Variables

5-3 Binomial Probability Distributions

Section 5-1
Review and Preview



Review and Preview

This chapter combines the methods of descriptive statistics presented in Chapter 2 and 3 and those of probability presented in Chapter 4 to describe and analyze

probability distributions.

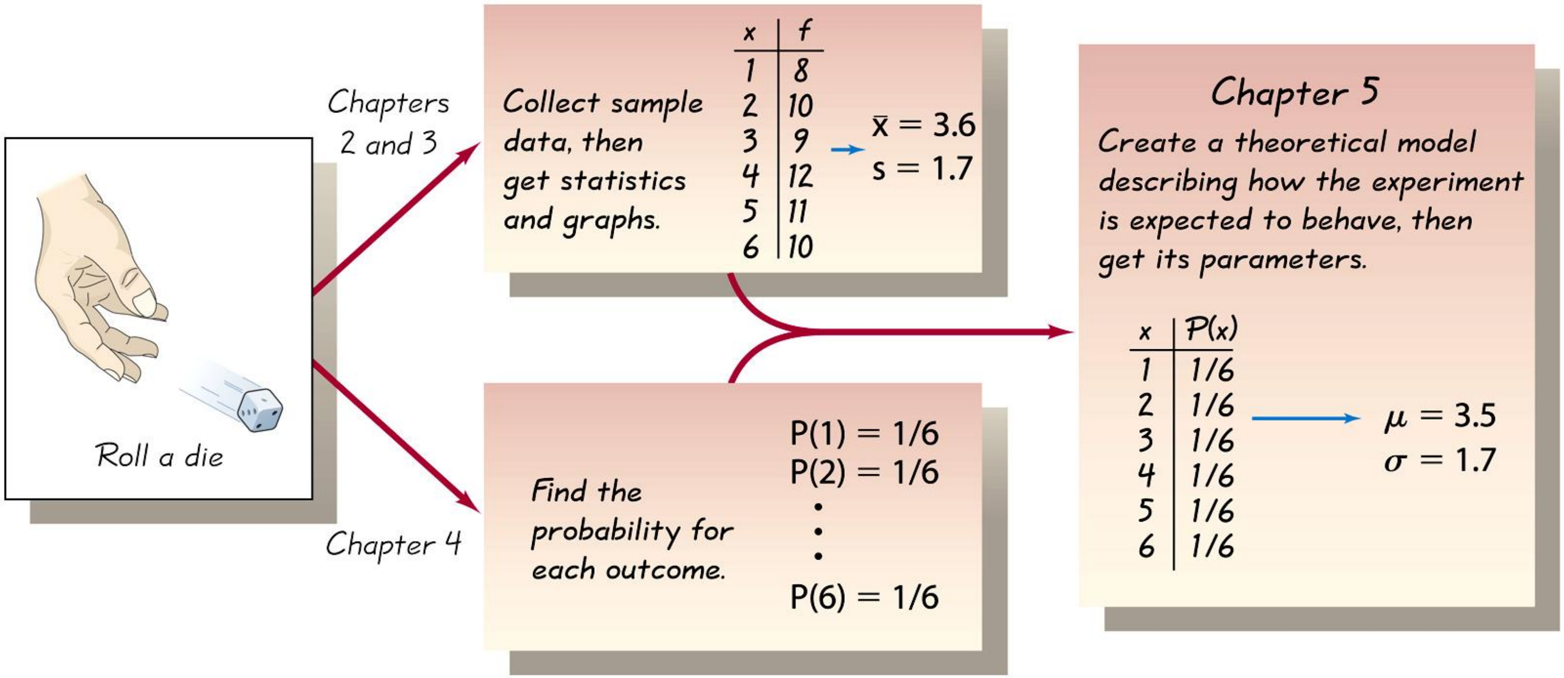
Probability Distributions describe what will probably happen instead of what actually did happen, and they are often given in the format of a graph, table, or formula.

Preview

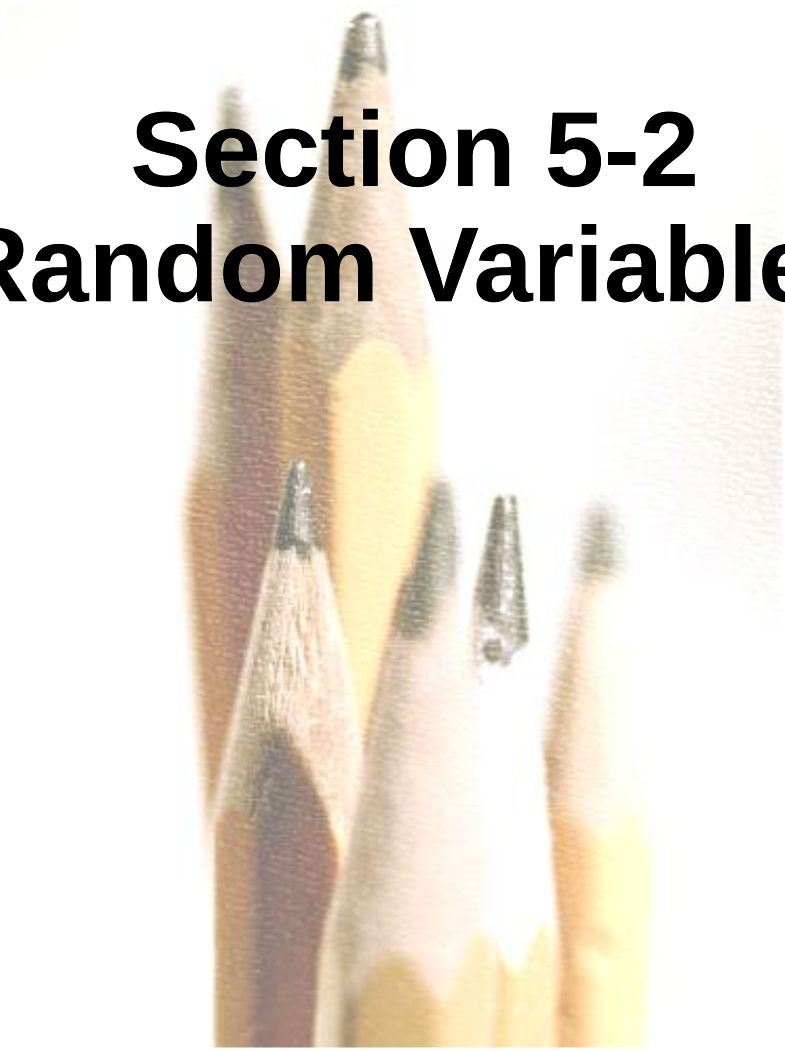
In order to fully understand probability distributions, we must first understand the concept of a random variable, and be able to distinguish between discrete and continuous random variables. In this chapter we focus on discrete probability distributions. In particular, we discuss binomial and Poisson probability distributions.

Combining Descriptive Methods and Probabilities

In this chapter we will construct probability distributions by presenting possible outcomes along with the relative frequencies we expect.



Section 5-2
Random Variables



Random Variable Probability Distribution

- **Random variable**
  a variable (typically represented by x) that has a single numerical value, determined by chance, for each outcome of a procedure
- **Probability distribution**
  a description that gives the probability for each value of the random variable; often expressed in the format of a graph, table, or formula

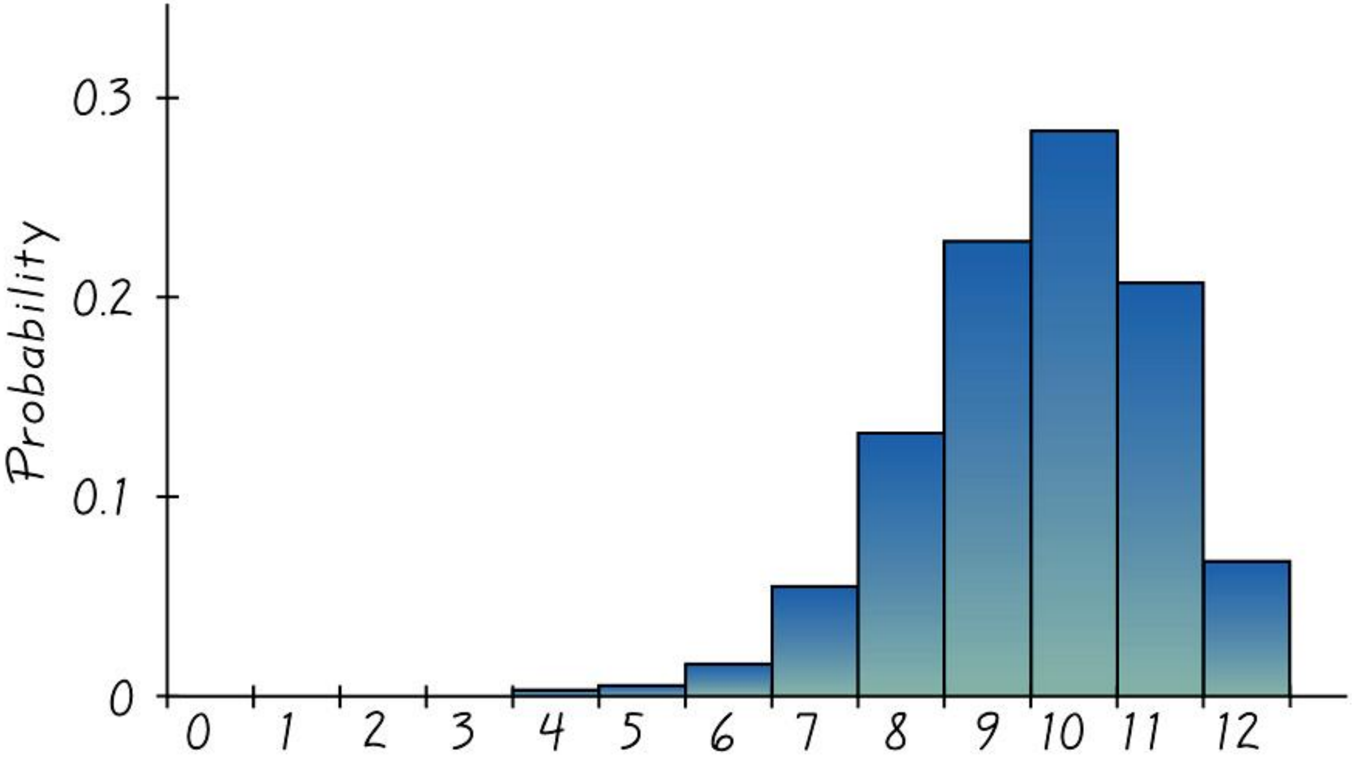
Discrete and Continuous Random Variables

- **Discrete random variable**
  either a finite number of values or countable number of values, where "countable" refers to the fact that there might be infinitely many values, but they result from a counting process

- **Continuous random variable**
  infinitely many values, and those values can be associated with measurements on a continuous scale without gaps or interruptions

Graphs

The probability histogram is very similar to a relative frequency histogram, but the vertical scale shows probabilities.



Probability Histogram for Number of Mexican-American Jurors Among 12

Requirements for Probability Distribution

$$\sum P(x) = 1$$

where *x* assumes all possible values.

$$0 \le P(x) \le 1$$

for every individual value of *x*.

Mean, Variance and Standard Deviation of a Probability Distribution

$$\mu = \Sigma\,[x \cdot P(x)]$$ **Mean**

$$\sigma^2 = \Sigma\,[(x-\mu)^2 \cdot P(x)]$$ **Variance**

$$\sigma^2 = \Sigma\,[x^2 \cdot P(x)] - \mu^2$$ **Variance (shortcut)**

$$\sigma = \sqrt{\Sigma\,[x^2 \cdot P(x)] - \mu^2}$$ **Standard Deviation**

Roundoff Rule for μ, σ, and σ^2

Round results by carrying one more decimal place than the number of decimal places used for the random variable x.

If the values of x are integers, round μ, σ, and σ^2 to one decimal place.

Identifying *Unusual* Results Range Rule of Thumb

According to the range rule of thumb, most values should lie within 2 standard deviations of the mean.

We can therefore identify "unusual" values by determining if they lie outside these limits:

Maximum usual value = $\mu + 2\sigma$

Minimum usual value = $\mu - 2\sigma$

Identifying Unusual Results Probabilities

Using Probabilities to Determine When Results Are Unusual

- **Unusually high**: x successes among n trials is an **unusually high** number of successes if $P(x \text{ or more}) \leq 0.05$.

- **Unusually low**: x successes among n trials is an **unusually low** number of successes if $P(x \text{ or fewer}) \leq 0.05$.

Expected Value

The expected value of a discrete random variable is denoted by E, and it represents the mean value of the outcomes. It is obtained by finding the value of $\Sigma\,[x \cdot P(x)]$.

$$E = \Sigma\,[x \cdot P(x)]$$

Examples

Pgs 208-211: 2, 6, 8, 13, 16, 26, 29