

Math 176: Cumulative Semester Review

☺ A ton of Practice Problems ☺

1.

a. $\mathbb{R}, x \neq 2$

b. $\mathbb{R}, x \neq 1$

c. $11/5$

d. 0

e. 3

f. No

g. $\frac{a+h+4}{a+h-2}$

h. 1

i. 5

j. $(-\infty, -1.2) \cup (1.2, \infty)$

k. $-11/3$

2. SEE GRAPH →

3. SEE GRAPH →

4. SEE GRAPH →

5. 8 in.

6. $k^{-1}(x) = \sqrt[3]{x-2} - 1$

7. $f(x) = (x+2)^2 - 3$ has min of -3

8.

a. shift left 2

b. shift down 3

c. reflected about the x-axis

d. reflected about $y = x$

9. building: $h(0)=32$ ft

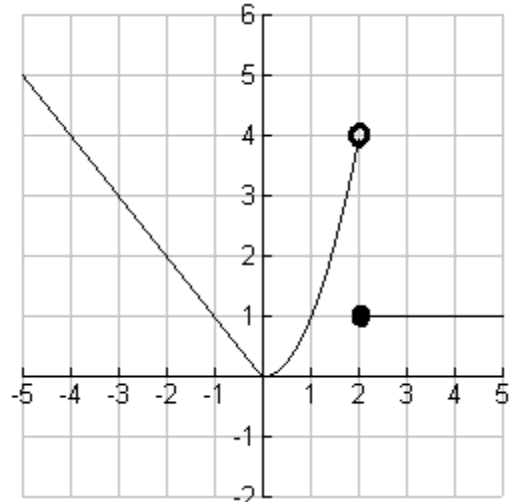
max height = $h(1.5)= 68$ ft

hits ground at approx. 3.56 sec

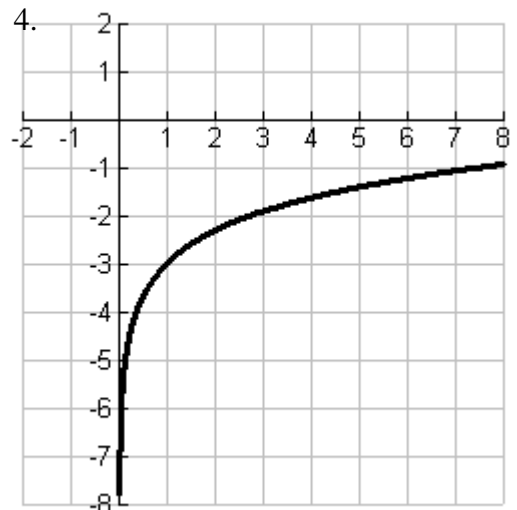
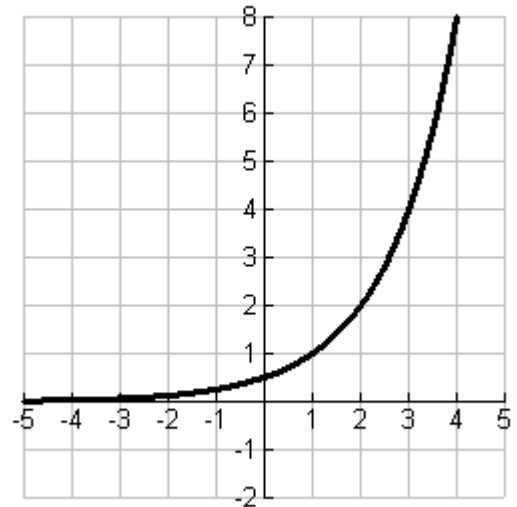
10. Graph and use vertical and horizontal line test. a. yes

b. no

2.



3.



11.

a. end behavior like x^5 , as $x \rightarrow \infty, P(x) \rightarrow \infty$
as $x \rightarrow -\infty, P(x) \rightarrow -\infty$

b. 5

c. Up to 4 (P(x) has only 2)

d. $\pm 1, \pm 2, \pm 4$

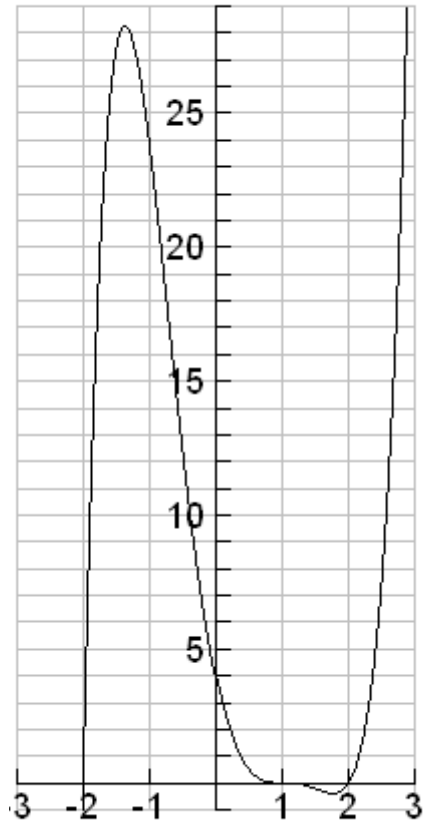
e. Zeros @ $x = -2, 1$ (mult. 3), 2

f. D: \mathbb{R} R: \mathbb{R}

g. Zeros @ $x = -2, 1$ (mult. 3), 2

h. $P(x) = (x+2)(x-1)^3(x-2)$

i. SEE GRAPH \rightarrow



12. $4x^2 - 12x - 34 + \frac{92}{x+3}$

13. $P(1) = 5$

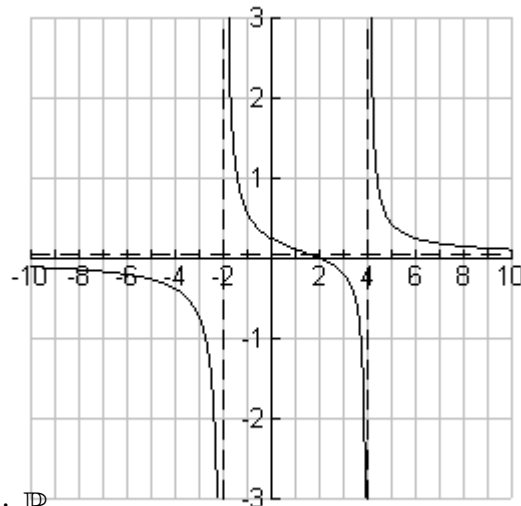
14.

a. $-46 - 22i$

b. $\frac{34}{25} - \frac{13}{25}i$

c. $1 + i$

d. $4 + i$



15. SEE GRAPH \rightarrow

D: $\mathbb{R}, x \neq 4, -2$ R: \mathbb{R}

16.

a. 3

d. 3

g. $3/2$

b. 7

e. -3

h. 30

c. 12

f. 0

17. $x = \frac{\ln 20}{4 \ln 5} \approx 0.465$

18. $x = \frac{10}{7} \approx 1.429$

19. $x = 2$

20. a. $A(t) = P\left(1 + \frac{r}{n}\right)^{nt} = 12,000\left(1 + \frac{.035}{12}\right)^{12t}$ b. $20,000 = 12,000\left(1 + \frac{.035}{12}\right)^{12t}$
 $A(3) = \$13,326.49$ $t = 14.6 \text{ years}$

21.

a. $-\frac{\sqrt{2}}{2}$ b. $\frac{\sqrt{3}}{2}$ c. $\frac{2\sqrt{3}}{3}$ d. -1 e. Undefined

f. -0.64941 g. $\sin(60^\circ + 45^\circ) = \frac{\sqrt{6} + \sqrt{2}}{2}$ h. $-\frac{\pi}{6}$ i. $-\sqrt{3}$

j. 0 k. $\frac{1}{2}$ l. $\cos\left(\frac{1}{2} \cdot \frac{\pi}{4}\right) = \sqrt{\frac{1 - \frac{\sqrt{2}}{2}}{2}} = \frac{\sqrt{2 - \sqrt{2}}}{2}$

22.

a. 23.09 c. 23.29 e. 87.41°
b. 37.87° d. 57.44

23. $-\frac{\sqrt{85}}{6}$

24. amp = 5, period = $\frac{2\pi}{3}$, phase shift: right $\frac{\pi}{3}$

25. $Area_{\text{sector}} - Area_{\text{triangle}} = \frac{1}{2}(9^2)\left(76 \cdot \frac{\pi}{180}\right) - \frac{1}{2}(9 \cdot 9)\sin 76^\circ = 14.4 \text{ sq. units}$

26. 73.9 feet

27. a. $LHS = 1 + \tan x \tan \frac{x}{2} = 1 + \frac{\sin x}{\cos x} \cdot \frac{1 - \cos x}{\sin x} = 1 + \frac{1 - \cos x}{\cos x} = 1 + \frac{1}{\cos x} + \frac{-\cos x}{\cos x}$
 $= 1 + \sec x - 1 = \sec x = RHS$

b. $LHS = \sin^2 x \cot^2 x + \cos^2 x \tan^2 x = \sin^2 x \frac{\cos^2 x}{\sin^2 x} + \cos^2 x \frac{\sin^2 x}{\cos^2 x} =$
 $= \cos^2 x + \sin^2 x = 1 = RHS$

c. $LHS = \frac{\sin 2x}{\sin x} - \frac{\cos 2x}{\cos x} = \frac{2\sin x \cos x}{\sin x} - \frac{2\cos^2 x - 1}{\cos x} = 2\cos x - \frac{2\cos^2 x}{\cos x} + \frac{1}{\cos x}$
 $= 2\cos x - 2\cos x + \sec x = \sec x = RHS$

28. $x = \pi n, \frac{\pi}{6} + 2\pi n, \frac{5\pi}{6} + 2\pi n$

29. $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$

30. $1 + i\sqrt{3} = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

31. $8 - 8i$

32. $|\mathbf{u}| = \sqrt{208}, |\mathbf{v}| = \sqrt{13}, \mathbf{u} + \mathbf{v} = \langle -5, 10 \rangle, \mathbf{u} - \mathbf{v} = \langle -11, 14 \rangle, 3\mathbf{u} = \langle -24, 36 \rangle, \frac{1}{4}\mathbf{u} - 3\mathbf{v} = \langle -11, 9 \rangle,$

and $\mathbf{u} \cdot \mathbf{v} = -48. \theta = 157.4^\circ$

33. $(-1 + \sqrt{3}, 1 + \sqrt{3})$ and $(-1 - \sqrt{3}, 1 - \sqrt{3})$

34. $(1, 1, 2)$

35. a. $\begin{bmatrix} 15 & 18 \\ 12 & -1 \end{bmatrix}$

b. DNE

c. 0

d. $\begin{bmatrix} 1 & 2/3 \\ 0 & 1/3 \end{bmatrix}$

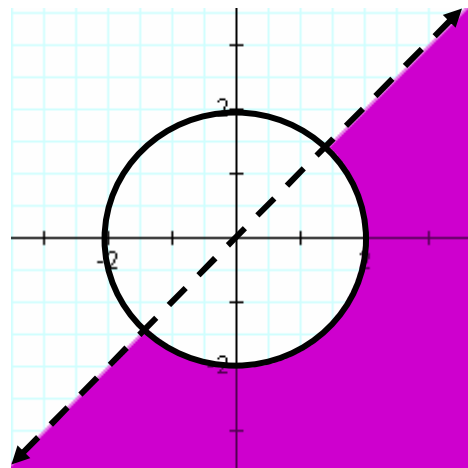
e. $\begin{bmatrix} 1 & -8 \\ 0 & 9 \end{bmatrix}$

f. DNE

g. -11

36. $\frac{9}{5(x-2)} + \frac{1}{5(x+3)}$

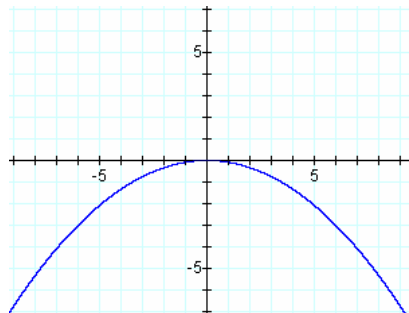
37. SEE GRAPH → (solution is shaded region)



38. focus: $(0, -3)$

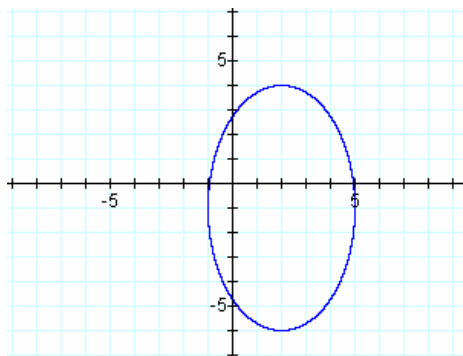
directrix: $y = 3$

SEE GRAPH →

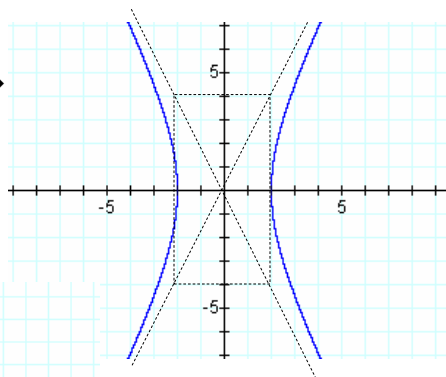


39. foci: $(2, 3)$ & $(2, -5)$ eccentricity: 0.8

SEE GRAPH →



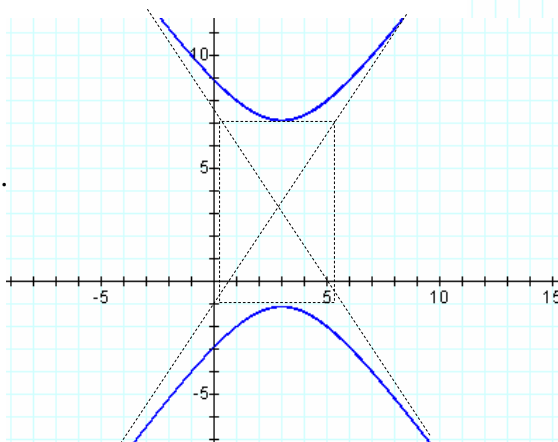
40. foci: $(0, \pm\sqrt{20})$ asymptotes: $y = \pm\frac{1}{2}x$ SEE GRAPH→



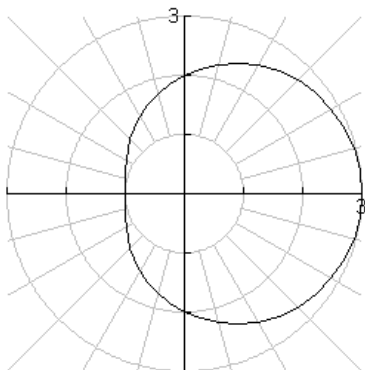
41. SEE GRAPH→

This equation is not pretty..

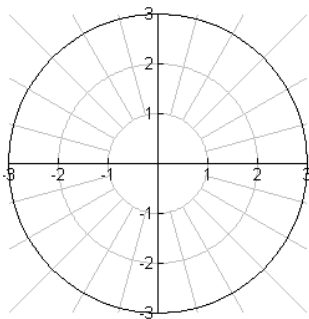
$$\frac{(y-3)^2}{17} - \frac{(x-3)^2}{(17/2)} = 1$$



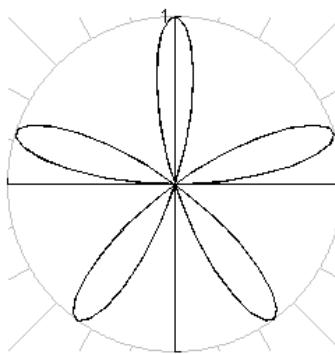
42. a.



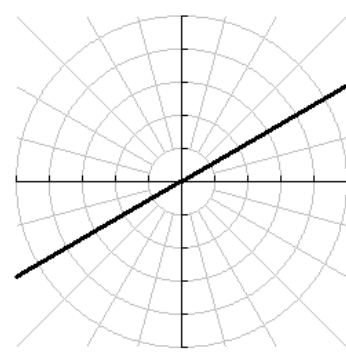
b.



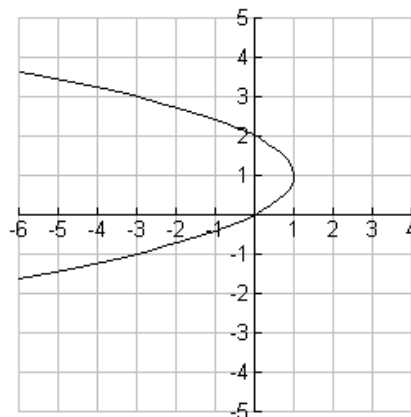
c.



d.



43. rectangular-coordinate equation: $x = 1 - (y-1)^2$ SEE GRAPH→



44. $x = 1 + 3t$
 $y = 3 + t$

45. a. $1, -\frac{1}{4}, \frac{1}{9}, -\frac{1}{16}, \frac{1}{25}, \dots$

b. $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \dots$

c. $5, 10, 25, 70, 115, \dots$

46. a. *arith*: $a_n = 4 + (n-1)6$, $a_6 = 34$

b. *neither*: $a_n = a_{n-1} + n - 1$, $a_6 = 19$

c. *geom*: $a_n = -2\left(\frac{-1}{2}\right)^{n-1}$, $a_6 = \frac{1}{16}$

d. *geom*: $a_n = 3(3i)^{n-1}$, $a_6 = 144i$

47. a. 349 b. 104 c. 4,194,300

$$n=1: 1 = \frac{1(3(1)-1)}{2} = 1$$

$$\text{Assume } n=k: 1+4+7+\dots+(3k-2) = \frac{k(3k-1)}{2}$$

$$\text{Prove } n=k+1: 1+4+7+\dots+(3k-2) + (3(k+1)-2) = \frac{(k+1)(3(k+1)-1)}{2}$$

48.

$$\underbrace{1+4+7+\dots+(3k-2)}_{\frac{k(3k-1)}{2}} + (3(k+1)-2) = \frac{k(3k-1)}{2} + (3(k+1)-2)$$

$$= \frac{3k^2 - k}{2} + \frac{2(3k+1)}{2} = \frac{3k^2 - k + 6k + 2}{2} = \frac{3k^2 + 5k + 2}{2} = \frac{(3k+2)(k+1)}{2} = \frac{(k+1)(3(k+1)-1)}{2}$$

so, by mathematical induction, the statement is true for all natural numbers, n.

49. $(x+2y)^5 = x^5 + 5x^4(2y) + 10x^3(2y)^2 + 10x^2(2y)^3 + 5x(2y)^4 + (2y)^5$
 $= x^5 + 10x^4y + 40x^3y^2 + 80x^2y^3 + 80xy^4 + 32y^5$

50. a. 15 b. 16 c. $\frac{n(n-1)}{2}$