Formula Sheet: Electricity and Magnetism

Coulomb's law

$$\vec{F}_{1,2} = \frac{kq_1q_2}{r_{1,2}^2}\hat{r}_{1,2}$$

The force on charge 1 due to charge 2. The unit vector points from charge 2 to charge 1

Electric Fields

$$\vec{F}(\text{on } q_0) = q_0\vec{E}$$

Field of a point charge $\vec{E} = \frac{kq}{r^2}\hat{r}$

Principle of superposition $\vec{E}_{net} = \sum_{i=1}^{n}\vec{E}_i$

Field from an infinitesimal charge element. $d\vec{E} = \frac{kdq}{r^2}\hat{r}$

Field on either side of an infinite charged plane $|E| = \frac{\sigma}{2\varepsilon_0}$

Discontinuity at the surface of charged plane $|\Delta E| = \frac{\sigma}{\varepsilon_0}$

Gauss's law

Flux: defined: $\phi_{net} = \oint_S \vec{E}\cdot\hat{n}dA = \oint_S E_n dA$

Gauss's Law: $\phi_{net} = 4\pi k q_{enclosed}$

$$= \frac{q_{enclosed}}{\varepsilon_0}$$

Electric potential

$$V(r) = \frac{kq}{r}$$

The potential for a point charge or outside a spherically symmetric charge distribution, with V=0 at infinity.

$$dV = \frac{kdq}{r}$$

potential from an infinitesimal charge element.

Potential calculated from the electric field

$$dV = -\vec{E}\cdot d\vec{\ell} \text{ and } -\frac{dV}{d\ell} = E_{\tan}$$

$$\Delta V = V_b - V_{ref} = -\int_{ref}^{b}\vec{E}\cdot d\vec{\ell}$$

Constants

$e = 1.602 \times 10^{-19}$ C (charge on e^- or p^+)

$1\text{eV} = 1.602 \times 10^{-19}$ J

$m_e = 9.11 \times 10^{-31}$ kg

$m_p = 1.67 \times 10^{-27}$ kg

$k = 1/(4\pi\varepsilon_o) = 8.99 \times 10^9$ N m^2/C^2

$\varepsilon_o = 8.85 \times 10^{-12}$ F/m (or $C^2/N\ m^2$)

$\mu_o = 4\pi \times 10^{-7}$ T m/A

$1\text{ T} = 10^4$ G

$g = 9.8\ m/s^2$

Differential area element: *dA*

dA = (circumference)x(thickness)

ring: $dA = 2\pi r\, dr$

Differential volume elements: *dv*

dv = (surface area)x(thickness)

thin sheet: $dv = A\, dy$

thin cylindrical shell: $dv = 2\pi r L\, dr$

thin spherical shell: $dv = 4\pi r^2\, dr$

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Potential Energy

$\Delta U = q_0 \Delta V$ for q_0 moving through ΔV

$U = \sum_{i=1}^{n} q_i V_i$; for a group of point charges bringing in each charge sequentially

$U = \frac{1}{2} QV$ for a conductor at potential V

Energy stored in a capacitor:

$$U = \tfrac{1}{2} CV^2 = \tfrac{1}{2} QV = \tfrac{1}{2} \frac{Q^2}{C}$$

Energy density of electric fields

$$u_e = \tfrac{1}{2} \varepsilon_0 E^2$$

Capacitors

$$C = \frac{Q}{V} \qquad \text{parallel plate}: C = \frac{\varepsilon_0 A}{d}$$

Capacitors in a circuit

parallel: $C_{equiv} = C_1 + C_2 + \ldots = \sum_{i=1}^{n} C_i$

series:

$$C_{equiv} = \frac{1}{1/C_1 + 1/C_2 + \ldots} = \frac{1}{\sum_{i=1}^{n} \frac{1}{C_i}}$$

DC Circuits

$$I = \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt} = qnAv_d$$

$$R = \rho \frac{L}{A}$$

Resistors in a circuit

series: $R_{equiv} = R_1 + R_2 + \ldots = \sum_{i=1}^{n} R_i$

parallel:

$$R_{equiv} = \frac{1}{1/R_1 + 1/R_2 + \ldots} = \frac{1}{\sum_{i=1}^{n} \frac{1}{R_i}}$$

Ohm's Law

$V = IR$

Power dissipated in a resistor

$$P = IV = I^2 R = \frac{V^2}{R}$$

Kirchhoff's rules

(1) at a junction: $\Sigma I_{in} = \Sigma I_{out}$

(2) around a closed loop: $\Sigma \Delta V = 0$

Time dependent circuits

($\mathcal{E}$ is the battery voltage (or other source of emf))

charge on a capacitor in an RC circuit:
You can derive *I(t)*and *V(t)* from *Q(t)*

Time constant: $\tau_{RC} = RC$

charging : $Q(t) = \mathcal{E} C\left(1 - e^{-t/\tau_{RC}}\right)$

discharging : $Q(t) = Q_o e^{-t/\tau_{RC}}$

current in an inductive (LR) circuit:
You can derive $V_R(t)$ from *I(t)*

Time constant : $\tau_{LR} = \frac{L}{R}$

When connecting the source of emf to the circuit:

$$I(t) = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau_{LR}}\right)$$

When disconnecting the source of emf from the circuit:

$$I(t) = I_o e^{-t/\tau_{LR}}$$

See the next page for magnetic energy.

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Magnetic Force

$\vec{F}_B = q\vec{v}\times\vec{B}$ [$\vec{F}_{Lorentz} = q(\vec{E}+\vec{v}\times\vec{B})$]

$d\vec{F}_B = Id\vec{l}\times\vec{B}$ $\vec{F} = I\vec{L}\times\vec{B}$

Magnetic Torques on current loops

Magnetic moment: $\vec{\mu} = NIA\hat{n}$

$\tau = NIAB\sin(\theta)$ $\vec{\tau} = \vec{\mu}\times\vec{B}$

potential energy of a current loop

$U = -\vec{\mu}\cdot\vec{B}$

Magnetic Fields

$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2}$ (point charge, not in the book)

$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{l}\times\hat{r}}{r^2}$ Biot – Savart Law

In the center of a single current loop of radius R

$B_{loop} = \frac{\mu_o}{4\pi}\frac{2\pi I}{R}$

On the axis of a single current loop of radius R

$B_{loop}(x) = \frac{\mu_o}{4\pi}\frac{2\pi R^2 I}{\left(x^2+R^2\right)^{3/2}}$ (x = 0 in the center)

Inside a long solenoid

$B_x = \mu_0 nI$ (n is loops/m = N/L)

Due to a very long straight wire

$B = \frac{\mu_0}{4\pi}\frac{2I}{r}$

Due to a short straight segment

$B = \frac{\mu_0}{4\pi}\frac{I}{r}(\sin\theta_2 - \sin\theta_1)$

Ampere's Law

$\oint_C \vec{B}\cdot d\vec{l} = \mu_0 I_{encircled}$

Induction
(see previous page for LR circuits)

Flux in a single loop

$\Phi_B = \int_S \vec{B}\cdot\hat{n}dA = BA\cos(\theta)$

Faraday's law (for one loop)

$\mathcal{E} = -\frac{d\Phi_B}{dt}$

Self inductance and self-induced emf

$L = N\Phi_B / I$ $\mathcal{E} = -L\frac{dI}{dt}$

Mutual inductance and induced emf

$M = N_2\Phi_{21}/I_1 = N_1\Phi_{12}/I_2$ $\mathcal{E} = -M\frac{dI}{dt}$

Here Φ_{21} means the flux in #2 due to #1 and Φ_{12} means the flux in #1 due to #2

Magnetic Energy

$U_m = \frac{1}{2}LI^2$ stored in an inductor

Remember, power is $\frac{dU}{dt}$

$u_m = \frac{1}{2}\frac{B^2}{\mu_0}$ energy density in a B field