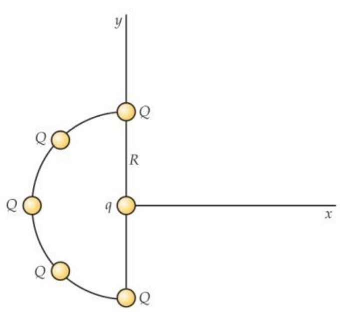
Physics 240, Review for Test 2

The test will cover Coulomb's law, electric fields and potentials. You will need to calculate fields and potentials for point charges. You will need to calculate fields by integrating relatively simple extended charges (for example, straight lines or arcs). You will need to calculate the motion of charges in a field. You need to be able to use Gauss's law for symmetric situations with a non-constant charge density. You will also need to calculate ΔV by integrating the field as a function of position and to calculate the field by differentiating the potential ($\Delta V = V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{\ell}$ and $-\frac{dV}{d\ell} = E_{\tan}$). You may have to draw field lines and equipotentials. You will need to understand the properties of conductors, specifically the field inside and at the surface. You should also be able to solve mechanics problems involving either the Coulomb law and charged particles or fields and charged particles. You should also be able to solve simple problems in involving conservation of energy. No examples of these are given here, but we had some <u>force</u> and <u>torque</u> problems and also conservation of energy problems on the homework. There could be conceptual short answer questions. **These problems do not exhaust the possibilities of material on the test.**

There are a few short topics in these chapters we didn't cover explicitly, so they won't be on the test: potential energy of an electric dipole in a field (part of 22-7), potential due to continuous charge distributions (24-5), and electric potential energy of a *system* of point charges (24-7) (that is NOT referring to energy conservation).

(1) Calculate the field and potential at the center of the semi-circle of point charges shown below. (There is a charge shown there, but ignore it for this calculation).



Answer: $\vec{E} = E_x \hat{i} = \frac{kQ}{R^2}(\sqrt{2}+1)\hat{i}$

$$V = \frac{5kQ}{R}$$

If the problem were not symmetric, you would have to work with both x and y components of the field. Redo the calculation for any two of the charges in the semi-circle.

Answer for the upper two charges: $\vec{E} = \frac{kQ}{R^2}\left[\frac{\sqrt{2}}{2}\hat{i} - \frac{2+\sqrt{2}}{2}\hat{j}\right]$

(2) A quarter of a circle of radius R is charged uniformly with charge Q. Calculate the field and potential at the center of the circle. How would you do this if the charge density were not uniform? Note: integration of charges to find potential is not on the test.

Answer: $E = \frac{2\sqrt{2}}{\pi}\frac{kQ}{R^2}, \theta = -45^\circ$

$$V = \frac{kQ}{R}$$

(3) A charged rod of length L lies on the x axis. The left edge of the rod is at the origin. The right edge of the rod is at $x = L$. The charge density on the rod varies as $\lambda(x) = \lambda_0 x$. Calculate the **field** at X_p where $X_p > L$. Calculate by integration of the field of charge elements dq. (If this were a test problem you would get the majority of points for setting up the integral correctly).

Answer: $E(x) = k\lambda_0\left[\ln\left(\frac{X_p - L}{X_p}\right) - \frac{LX_p}{X_p(X_p - L)}\right]$

(4) An infinitely long nonconducting cylinder of radius R carries a nonuniform volume charge density of $\rho(r) = ar$. Calculate the charge per unit length of the cylinder. Calculate the potential *and* the field as functions of r. Note, for the potential, you can choose an arbitrary reference point and calculate ΔV, but if you try to use infinity it blows up.

Answer: $\lambda = 2\pi a R^3/3$

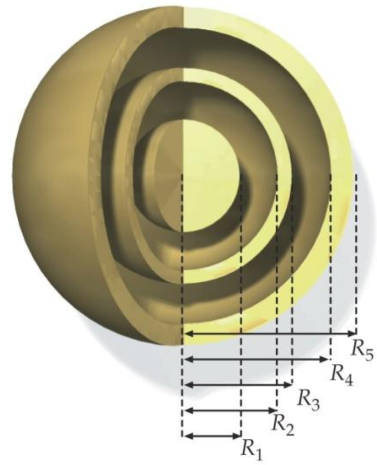
$$r > R:\ E_r(r) = \frac{aR^3}{3\varepsilon_\circ}\frac{1}{r};\ r < R:\ E_r(r) = \frac{ar^2}{3\varepsilon_\circ}$$

$$r > R:\ \Delta V = V(r) - V(r_{ref}) = \frac{aR^3}{3\varepsilon_\circ}\ln\left(\frac{r_{ref}}{r}\right)$$

$$r < R:\ \Delta V = V(r) - V(r_{ref}) = \frac{a}{3\varepsilon_\circ}\left[\frac{R^3 - r^3}{3} + R^3\ln\left(\frac{r_{ref}}{R}\right)\right]$$

That's probably too much for the test, but if you can do it, you ought to be ready for the test! Plus, I won't give you anything that blows up at unexpected places.

(5) Consider the three concentric metal (conducting) spheres shown on the right. Sphere one is a solid, with radius R. Sphere two is hollow, with inner radius R_2 and outer radius R_3. Sphere three is hollow, with inner radius R_4 and outer radius R_5. A negative charge $-Q_0$ is placed on sphere one and a positive charge $+Q_0$ is placed on sphere three. How do the charges distribute themselves? Where is the field zero? Which way does the field point in regions where it is not zero (sketch it)? Calculate the potential *and* the field as functions of *r*. Graph them. Use Gauss's law to calculate the field and calculate the potential by integrating E from infinity. It's easier to get the potential by summing the potentials inside and outside charged spheres, but we are not covering that in great detail.



Answer: $r < R_1:$ $\quad E_r = 0, \quad V(r) = -kQ\left[\left(\frac{1}{R_3}+\frac{1}{R_1}\right)-\left(\frac{1}{R_4}+\frac{1}{R_2}\right)\right]$

$$R_1 < r < R_2 : E_r = -\frac{kQ}{r^2}, \; V(r) = -kQ\left[\left(\frac{1}{R_3}+\frac{1}{r}\right)-\left(\frac{1}{R_4}+\frac{1}{R_2}\right)\right]$$

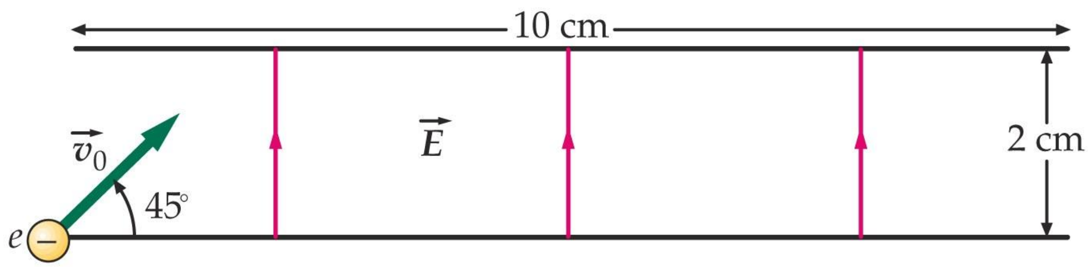
$$R_2 < r < R_3 : \; E_r = 0, \qquad V(r) = -kQ\left(\frac{1}{R_3}-\frac{1}{R_4}\right)$$

$$R_3 < r < R_4 : E_r = -\frac{kQ}{r^2}, \quad V(r) = -kQ\left(\frac{1}{r}-\frac{1}{R_4}\right)$$

$$R_4 < r : \qquad E_r = 0, \qquad V(r) = 0$$

That's definitely too much for a test, but you could get one with fewer parts!

(6) An electron starts at the position shown below with an initial speed of $v_0 = 5 \times 10^6$ m/s at an angle of 45° to the *x* axis. The electric field is in the positive *y* direction and has a magnitude of 3.5×10^3 N/C. On which plate and at what location will the electron strike?



Answer: lower plate, $x = 4.07$ cm.