WEEK 1
LIMITS

Calculus is a powerful branch of mathematics with a wide range of applications, including curve sketching, optimization of functions (maximizing and minimizing), analysis of rates of change, and computation of area and volumes. We will begin our study with an important concept called the "limit".

(1.1) Limits: A Numeric and Graphical Approach

The limit process involves examining the behavior of a function *f(x)* as *x* approaches a number *a*, that may or may not be in its domain of *f*.

Limit – the ultimate value of a function as x (or other variable) approaches a specific value.

Notation: $\lim_{x \to a} f(x) = L$

"the limit of *f(x)* as *x* approaches *a*"

The limit must approach the same number on both sides. We call this left and right hand limits.

Right-side limit: $\lim_{x \to a^+} f(x) = L$

Left-side limit: $\lim_{x \to a^-} f(x) = L$

Examples:

1) $\lim_{x \to 4}(2x+1)$

2) $\lim_{x \to 3} \frac{x}{x-3}$

A common type of function used in business and other situations is the piecewise function. The piecewise function is a function which is defined by multiple sub-functions. Each sub-function is applied to a certain interval of the main functions domain.

Example: $G(x) = \begin{cases} -x+3 & for \quad x<1 \\ x & for \quad x \geq 1 \end{cases}$ Find $\lim_{x\to 2^-} G(x)$, $\lim_{x\to 2^+} G(x)$, and $\lim_{x\to 2} G(x)$

(1.2) Algebraic Limits and Continuity

RULES OF LIMITS:

1) $\lim_{x\to a} c = c$

2) $\lim_{x\to a} \left[f(x)\right]^n = \left[\lim_{x\to a} f(x)\right]^n = L^n$ AND $\lim_{x\to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x\to a} f(x)} = \sqrt[n]{L}$

3) $\lim_{x\to a}[f(x) \pm g(x)] = \lim_{x\to a} f(x) \pm \lim_{x\to a} g(x)$

4) $\lim_{x\to a}[f(x) \cdot g(x)] = \lim_{x\to a} f(x) \cdot \lim_{x\to a} g(x)$

5) $\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$ as long as $g(x) \neq 0$

6) $\lim_{x\to a}\left[cf(x)\right] = c \cdot \lim_{x\to a} f(x)$

- **You can evaluate limits with direct substitution. If you get an undefined expression, rewrite the expression and substitute again.**

Examples: Find the following limits:

1) $\lim_{x\to 2} x^2 + 3x - 5$

2) $\lim_{x\to 3} \frac{2x^2 - 5x - 3}{x - 3}$

3) $\lim_{h\to 0} \frac{2xh - 3h^2}{h}$