

## Forty Couples

Forty intelligent couples live on a remote island. In their society, three rules are rigidly adhered to:

1. Whenever a woman is unfaithful to her husband, the other thirty-nine men meet and commonly share this information.
2. No one ever tells a man that his wife has been unfaithful to him.
3. If a man ever goes to bed with his wife in the evening, knowing for sure that she has been unfaithful to him at some time in the past, he kills her before morning (and his action becomes public knowledge in the morning).

The actual state of “affairs” is that all forty wives have at some time in the past been unfaithful (although each has---to herself---forsworn further affairs). Each man therefore knows that there are at least thirty-nine unfaithful wives on the island (the wives of the other men), but is unsure about his wife’s fidelity. For years, everyone has been living happily (if a bit uneasily) in this setting.

One day, a missionary comes to the island for a visit. He stays for several weeks, during which he visits privately with each resident. On the day of his departure, as the eighty residents gather to watch him row away, he calls out, “I enjoyed my visit, but was disappointed to learn that there is at least one unfaithful wife on the island.” He then passes beyond earshot, before any questions can be asked of him. Of course, none of the husbands are surprised to hear his statement. And indeed, the next morning no wives are found to have been killed. Yet on the fortieth night after his departure, just when the wives are beginning to relax, all forty men kill their spouses.

a) Assuming that the 40 husbands are very intelligent, rational, and they all believe what the missionary said, how can they deduce on the 40<sup>th</sup> day that their wives are unfaithful?

(You might start by considering a simpler situation, in which only two wives on the island are unfaithful, and the two husbands kill their wives on the 2<sup>nd</sup> night after the missionary’s remark. After this use induction on the number of unfaithful wives.)

b) Clearly, the missionary’s last remark is not any news to any body, but what is new that leads the 40 husbands to kill all their wives on the 40<sup>th</sup> day?