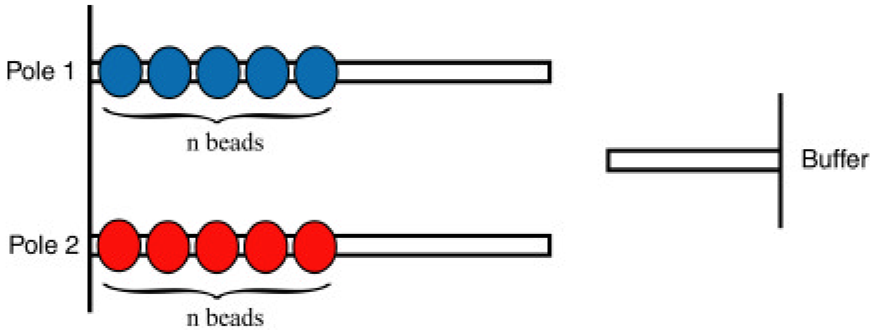
The problem of swapping beads.

Let $\boldsymbol{b}_{\mathbf{n}}$ be the minimum number of moves to swap n pairs of beads.

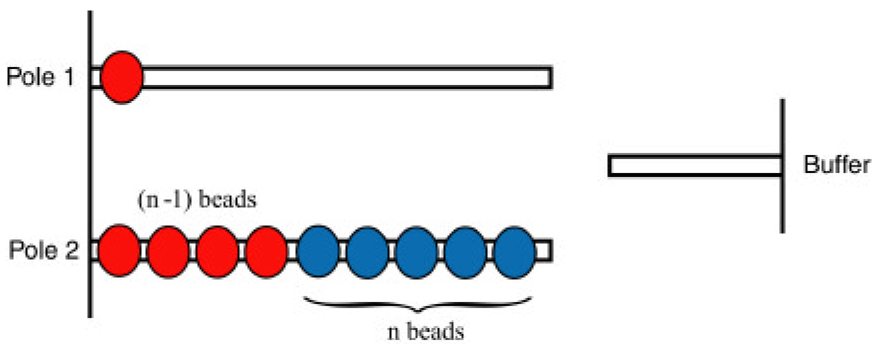
The rules are
1. only one bead can be move at one time.
2. the beads must be on either Pole 1, Pole 2, or the buffer.
3. the buffer can hold at most only two beads.

(we have to assume that both poles are long enough to hold $2n$ beads.)

In the beginning we have the following diagram

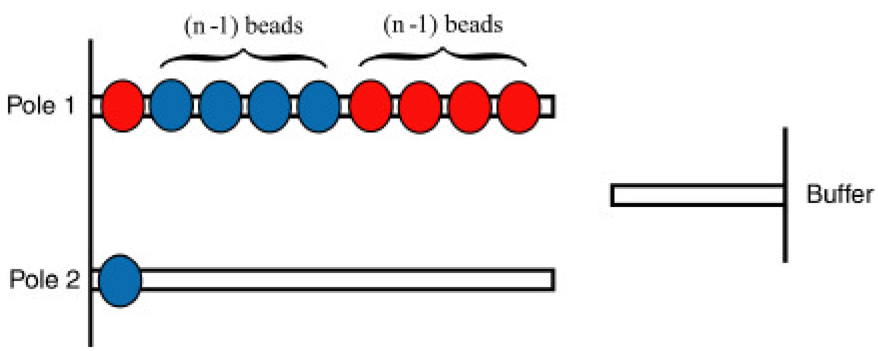


1st step: insert one red bead to the base of pole 1. The result is indicated below.



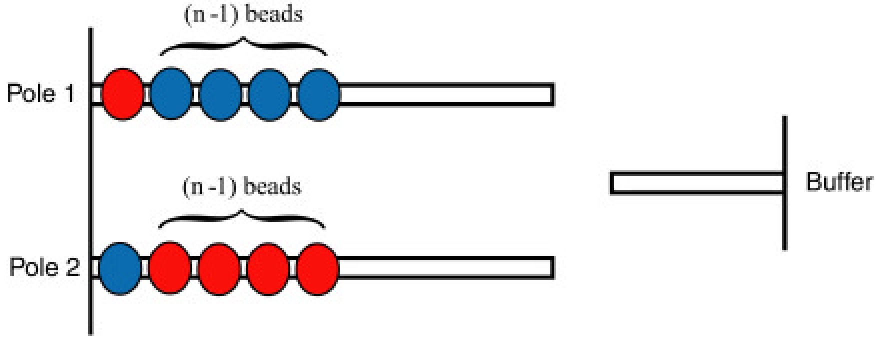
This requires at least 1 + 2n + 1 (= **2n + 2**) moves.

2nd step: insert one blue bead to the base of pole 2. The result is indicated below.



This requires at least **4(n-1) + 2** moves.

3rd step: move the (n-1) red beads back to Pole 2 so that we can use recursion.



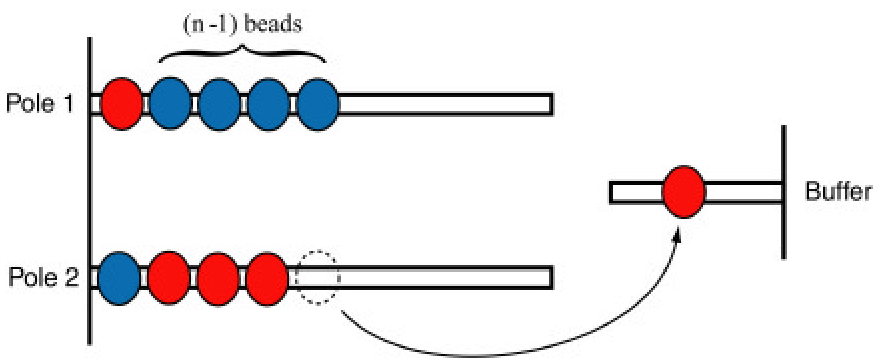
This requires at least **2(n-1)** moves.

4th step: swap the remaining (n-1) pairs of beads.

This requires of course by definition, $\boldsymbol{b}_{(n-1)}$ moves.

So at this point we can say that $\boldsymbol{b}_n \leq [2n + 2] + [4(n-1) + 2] + [2(n-1)] + \boldsymbol{b}_{(n-1)}$

However this is not the minimal number because in the first move of swapping the (n-1) pairs, we do the following.



So there is really no need to move all red beads to pole 2, we can simply leave one on the buffer. This can save us a total of 2 steps.

Therefore $\boldsymbol{b}_n = [2n + 2] + [4(n-1) + 2] + [2(n-1)] + \boldsymbol{b}_{(n-1)} - 2$, which can be simplified to

$$\boldsymbol{b}_n = \boldsymbol{b}_{(n-1)} + 8n - 4$$