The steps for each algebraic method for solving systems of two equations are given below. Note that in both methods, we find the value of one variable and then substitute to find the corresponding value of the other variable.

To Solve a System Using Substitution

1. Isolate a variable in one of the equations (unless one is already isolated).
2. Substitute for that variable in the other equation, using parentheses.
3. Solve for the remaining variable.
4. Substitute the value of the second variable in any of the equations, and solve for the first variable.
5. Form an ordered pair and check in the original equations.

To Solve a System Using Elimination

1. Write both equations in standard form.
2. Multiply both sides of one or both equations by a constant, if necessary, so that the coefficients of one of the variables are opposites.
3. Add the left sides and the right sides of the resulting equations. One variable should be eliminated in the sum.
4. Solve for the remaining variable.
5. Substitute the value of the second variable in any of the equations, and solve for the first variable.
6. Form an ordered pair and check in the original equations.

3.2 Exercise Set

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Concept Reinforcement *In each of Exercises 1–6, match the system listed with the choice from the column on the right that would be a subsequent step in solving the system.*

1. __(d)__ $3x - 4y = 6,$ $5x + 4y = 1$
2. __(e)__ $2x - y = 8,$ $y = 5x + 3$
3. __(a)__ $x - 2y = 3,$ $5x + 3y = 4$
4. __(f)__ $8x + 6y = -15,$ $5x - 3y = 8$
5. __(c)__ $y = 4x - 7,$ $6x + 3y = 19$
6. __(b)__ $y = 4x - 1,$ $y = -\frac{2}{3}x - 1$

a) $-5x + 10y = -15,$ $5x + 3y = 4$

b) The lines intersect at $(0, -1)$.

c) $6x + 3(4x - 7) = 19$

d) $8x = 7$

e) $2x - (5x + 3) = 8$

f) $8x + 6y = -15,$ $10x - 6y = 16$

For Exercises 7–54, if a system has an infinite number of solutions, use set-builder notation to write the solution set. If a system has no solution, state this.

Solve using the substitution method.

7. $y = 3 - 2x,$ $3x + y = 5$ $(2, -1)$
8. $3y + x = 4,$ $x = 2y - 1$ $(1, 1)$
9. $3x + 5y = 3,$ $x = 8 - 4y$ $(-4, 3)$
10. $9x - 2y = 3,$ $3x - 6 = y$ $(-3, -15)$
11. $3s - 4t = 14,$ $5s + t = 8$ $(2, -2)$
12. $m - 2n = 16,$ $4m + n = 1$ $(2, -7)$
13. $4x - 2y = 6,$ $2x - 3 = y$ $\{(x, y) \mid 2x - 3 = y\}$
14. $t = 4 - 2s,$ $t + 2s = 6$ No solution
15. $-5s + t = 11,$ $4s + 12t = 4$ $(-2, 1)$
16. $5x + 6y = 14,$ $-3y + x = 7$ $(4, -1)$

17. $2x + 2y = 2,$
$3x - y = 1$ $\left(\frac{1}{2}, \frac{1}{2}\right)$

18. $4p - 2q = 16,$
$5p + 7q = 1$ $(3, -2)$

19. $2a + 6b = 4,$
$3a - b = 6$ $(2, 0)$

20. $3x - 4y = 5,$
$2x - y = 1$ $\left(-\frac{1}{5}, -\frac{7}{5}\right)$

21. $2x - 3 = y,$
$y - 2x = 1$
No solution

22. $a - 2b = 3,$
$3a = 6b + 9$
$\{(a, b) | a - 2b = 3\}$

Solve using the elimination method.

23. $x + 3y = 7,$
$-x + 4y = 7$ $(1, 2)$

24. $2x + y = 6,$
$x - y = 3$ $(3, 0)$

25. $x - 2y = 11,$
$3x + 2y = 17$ $(7, -2)$

26. $5x - 3y = 8,$
$-5x + y = 4$ $(-2, -6)$

27. $9x + 3y = -3,$
$2x - 3y = -8$ $(-1, 2)$

28. $6x - 3y = 18,$
$6x + 3y = -12$ $\left(\frac{1}{2}, -5\right)$

29. $5x + 3y = 19,$
$x - 6y = 11$ $\left(\frac{49}{11}, -\frac{12}{11}\right)$

30. $3x + 2y = 3,$
$9x - 8y = -2$ $\left(\frac{10}{21}, \frac{11}{14}\right)$

31. $5r = 3s + 24,$
$3r + 5s = 28$ $(6, 2)$

32. $5x = 7y - 16,$
$2x + 8y = 26$ $(1, 3)$

33. $6s + 9t = 12,$
$4s + 6t = 5$ No solution

34. $10a + 6b = 8,$
$5a + 3b = 2$
No solution

35. $\frac{1}{2}x - \frac{1}{6}y = 10,$
$\frac{2}{5}x + \frac{1}{2}y = 8$ $(20, 0)$

36. $\frac{1}{3}x + \frac{1}{5}y = 7,$
$\frac{1}{6}x - \frac{2}{5}y = -4$ $(12, 15)$

37. $\frac{x}{2} + \frac{y}{3} = \frac{7}{6},$
$\frac{2x}{3} + \frac{3y}{4} = \frac{5}{4}$ $(3, -1)$

38. $\frac{2x}{3} + \frac{3y}{4} = \frac{11}{12},$
$\frac{x}{3} + \frac{7y}{18} = \frac{1}{2}$ $(-2, 3)$

Aha! **39.** $12x - 6y = -15,$
$-4x + 2y = 5$
$\{(x, y) | -4x + 2y = 5\}$

40. $8s + 12t = 16,$
$6s + 9t = 12$
$\{(s, t) | 6s + 9t = 12\}$

41. $0.3x + 0.2y = 0.3,$
$0.5x + 0.4y = 0.4$ $\left(2, -\frac{3}{2}\right)$

42. $0.3x + 0.2y = 5,$
$0.5x + 0.4y = 11$
$(-10, 40)$

Solve using any appropriate method.

43. $a - 2b = 16,$
$b + 3 = 3a$ $(-2, -9)$

44. $5x - 9y = 7,$
$7y - 3x = -5$ $\left(\frac{1}{2}, -\frac{1}{2}\right)$

45. $10x + y = 306,$
$10y + x = 90$ $(30, 6)$

46. $3(a - b) = 15,$
$4a = b + 1$ $\left(-\frac{4}{3}, -\frac{19}{3}\right)$

47. $6x - 3y = 3,$
$4x - 2y = 2$ $\{(x, y) | 4x - 2y = 2\}$

48. $x + 2y = 8,$
$x = 4 - 2y$ No solution

49. $3s - 7t = 5,$
$7t - 3s = 8$ No solution

50. $2s - 13t = 120,$
$-14s + 91t = -840$ $\{(s, t) | 2s - 13t = 120\}$

51. $0.05x + 0.25y = 22,$
$0.15x + 0.05y = 24$ $(140, 60)$

52. $2.1x - 0.9y = 15,$
$-1.4x + 0.6y = 10$ No solution

53. $13a - 7b = 9,$
$2a - 8b = 6$ $\left(\frac{1}{3}, -\frac{2}{3}\right)$

54. $3a - 12b = 9,$
$4a - 5b = 3$ $\left(-\frac{3}{11}, -\frac{9}{11}\right)$

In Exercises 55–58, determine which of the given viewing windows below shows the point of intersection of the graphs of the equations in the given system. Check by graphing.

a) $[-5, 5, -5, 5]$
b) $[25, 50, 0, 10]$
c) $[-20, 0, 0, 50]$
d) $[100, 200, 0, 100]$

55. The system of Exercise 51 (d)

56. The system of Exercise 44 (a)

57. The system of Exercise 45 (b)

58. The system of Exercise 42 (c)

TW **59.** Describe a procedure that can be used to write an inconsistent system of equations.

TW **60.** Describe a procedure that can be used to write a system that has an infinite number of solutions.

SKILL REVIEW

To prepare for Section 3.3, review solving problems using the five-step problem-solving strategy (Section 1.7).

Solve. [1.7]

61. *Energy Consumption.* With average use, a toaster oven and a convection oven together consume 15 kilowatt hours (kWh) of electricity each month. A convection oven uses four times as much electricity as a toaster oven. How much does each use per month? Toaster oven: 3 kWh; convection oven: 12 kWh
Source: Lee County Electric Cooperative

62. *Test Scores.* Ellia needs to average 80 on her tests in order to earn a B in her math class. Her average after 4 tests is 77.5. What score is needed on the fifth test in order to raise the average to 80? 90

63. *Real Estate.* After her house had been on the market for 6 months, Gina reduced the price to \$94,500. This was $\frac{9}{10}$ of the original asking price. How much did Gina originally ask for her house?
\$105,000

64. *Car Rentals.* National Car Rental rents minivans to a university for \$69 per day plus 30¢ per mile. An English professor rented a minivan for 2 days to take a group of students to a seminar. The bill was \$225. How far did the professor drive the van?
Source: www.nationalcar.com 290 mi

65. *Carpentry.* Anazi cuts a 96-in. piece of wood trim into three pieces. The second piece is twice as long as the first. The third piece is one-tenth as long as the second. How long is each piece?
First: 30 in.; second: 60 in.; third: 6 in.

66. *Telephone Calls.* Terri's voice over the Internet (VoIP) phone service charges \$0.36 for the first minute of each call and \$0.06 for each additional $\frac{1}{2}$ minute. One month she was charged \$28.20 for 35 calls. How many minutes did she use? 165 min

SYNTHESIS

TW **67.** Some systems are more easily solved by substitution and some are more easily solved by elimination. What guidelines could be used to help someone determine which method to use?

TW **68.** Explain how it is possible to solve Exercise 39 mentally.

69. If $(1, 2)$ and $(-3, 4)$ are two solutions of $f(x) = mx + b$, find m and b. $m = -\frac{1}{2}, b = \frac{5}{2}$

70. If $(0, -3)$ and $\left(-\frac{3}{2}, 6\right)$ are two solutions of $px - qy = -1$, find p and q. $p = 2, q = -\frac{1}{3}$

71. Determine a and b for which $(-4, -3)$ is a solution of the system

$$ax + by = -26,$$
$$bx - ay = 7.$$
$a = 5, b = 2$

72. Solve for x and y in terms of a and b:

$$5x + 2y = a,$$
$$x - y = b.$$
$\left(\frac{a + 2b}{7}, \frac{a - 5b}{7}\right)$

Solve.

73. $\dfrac{x + y}{2} - \dfrac{x - y}{5} = 1,$

$\dfrac{x - y}{2} + \dfrac{x + y}{6} = -2$ $\left(-\frac{32}{17}, \frac{38}{17}\right)$

74. $3.5x - 2.1y = 106.2,$

$4.1x + 16.7y = -106.28$ $(23.118879, -12.039964)$

Each of the following is a system of nonlinear equations. However, each is reducible to linear, since an appropriate substitution (say, u for $1/x$ and v for $1/y$) yields a linear system. Make such a substitution, solve for the new variables, and then solve for the original variables.

75. $\dfrac{2}{x} + \dfrac{1}{y} = 0,$

$\dfrac{5}{x} + \dfrac{2}{y} = -5$ $\left(-\frac{1}{5}, \frac{1}{10}\right)$

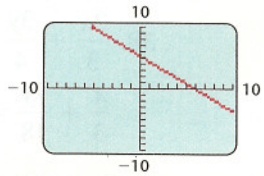
76. $\dfrac{1}{x} - \dfrac{3}{y} = 2,$

$\dfrac{6}{x} + \dfrac{5}{y} = -34$ $\left(-\frac{1}{4}, -\frac{1}{2}\right)$

TW **77.** A student solving the system

$$17x + 19y = 102,$$
$$136x + 152y = 826$$

graphs both equations on a graphing calculator and gets the following screen. The student then (incorrectly) concludes that the equations are dependent and the solution set is infinite. How can algebra be used to convince the student that a mistake has been made?



Try Exercise Answers: Section 3.2

7. $(2, -1)$ **11.** $(2, -2)$ **21.** No solution **23.** $(1, 2)$ **29.** $\left(\frac{49}{11}, -\frac{12}{11}\right)$ **31.** $(6, 2)$ **33.** No solution **35.** $(20, 0)$ **47.** $\{(x, y) | 4x - 2y = 2\}$