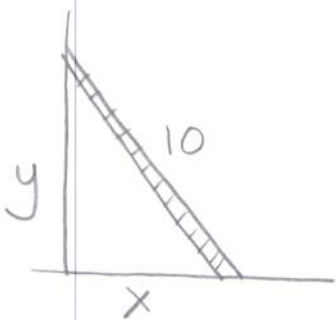


Math 180: Related Rates, 3.9 – Day 1

Strategy for Solving Related Rates Problems

1. Read the problem carefully.
2. Draw a diagram and label all known and unknown lengths or quantities. If certain lengths CHANGE with respect to time, they should be represented by a VARIABLE. If other lengths are FIXED over time, label those lengths with numbers.
3. Determine the RATE that you are trying to find, using derivative notation
For example: $\frac{dA}{dt}$, for the change in Area with respect to time.
Also, write down any instantaneous rates given in the problem. Think, are the rates positive or negative? If the length is increasing (+) rate, if decreasing (-) rate.
4. Write an equation that relates all of the quantities/variables of the problem. If necessary use the geometry of the situation to eliminate one of the variables by substitution.
5. Perform implicit differentiation with respect to time, t (using the chain rule).
6. Substitute the given information into the resulting equation and solve for the UNKNOWN RATE.

1. A ladder 10 ft long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 1 ft/s, how fast is the top of the ladder sliding down the wall when the bottom of the ladder is 6 ft from the wall?



Find $\frac{dy}{dt}$.

$$x^2 + y^2 = 10^2$$

$$\frac{d}{dt}[x^2 + y^2] = \frac{d}{dt}[100]$$

$$2x \cdot \frac{dx}{dt} + 2y \cdot \frac{dy}{dt} = 0$$

plug + chug:

$$2(6)(1) + 2(8)\left(\frac{dy}{dt}\right) = 0$$

$$16 \cdot \frac{dy}{dt} = -12$$

$$\frac{dy}{dt} = -\frac{3}{4} \text{ ft/sec}$$

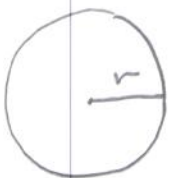
moment in time

$$x = 6 \text{ ft}$$

$$y = \sqrt{100 - 36} = \sqrt{64} = 8$$

$$\frac{dx}{dt} = 1 \text{ ft/s}$$

2. Suppose oil spills from a ruptured tanker and spreads in a circular pattern. If the radius of the oil spill increases at a constant rate of 1 m/s, how fast is the area of the spill increasing when the radius is 30 m?



Find $\frac{dA}{dt}$.

$$A = \pi r^2$$

$$\frac{d}{dt}(A) = \frac{d}{dt}[\pi r^2]$$

$$\frac{dA}{dt} = 2\pi r \cdot \frac{dr}{dt}$$

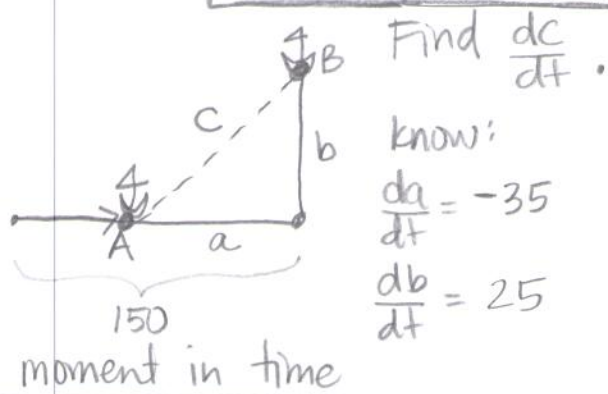
$$\frac{dA}{dt} = 2\pi(30)(1) = 60\pi \text{ m}^2/\text{s}$$

moment in time

$$r = 30 \text{ m}$$

$$\frac{dr}{dt} = 1 \text{ m/s}$$

3. At noon, ship A is 150 km west of ship B. Ship A is sailing east at 35 km/h and ship B is sailing north at 25 km/h. How fast is the distance between the ships changing at 4:00pm?



at 4:00 pm, 4 hrs have past,

$$a = 150 - 4(35) = 10 \text{ km}$$

$$b = 4(25) = 100 \text{ km}$$

$$c = \sqrt{10^2 + 100^2} = 10\sqrt{101} \text{ km}$$

$$a^2 + b^2 = c^2$$

$$\frac{d}{dt}[a^2 + b^2] = \frac{d}{dt}[c^2]$$

$$2a \cdot \frac{da}{dt} + 2b \cdot \frac{db}{dt} = 2c \cdot \frac{dc}{dt}$$

plug in:

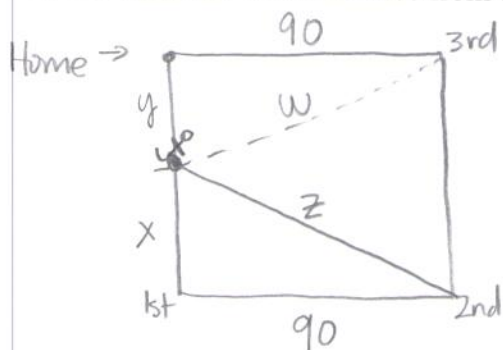
$$2(10)(-35) + 2(100)(25) = 2(10\sqrt{101}) \cdot \frac{dc}{dt}$$

$$\frac{dc}{dt} = \frac{4300}{20\sqrt{101}} = \boxed{21.39 \text{ km/hr}}$$

4. A baseball diamond is square with side 90 ft. A batter hits the ball and runs toward first base with a speed of 24 ft/s.

a. At what rate is his distance from second base decreasing when he is halfway to first base? Find $\frac{dz}{dt}$.

b. At what rate is his distance from third base increasing at the same moment? Find $\frac{dw}{dt}$.



moment in time

$$x = 45 \text{ ft}$$

$$z = \sqrt{45^2 + 90^2}$$

$$z = 45\sqrt{5}$$

$$\frac{dx}{dt} = -24 \text{ ft/s}$$

$$a) \quad x^2 + 90^2 = z^2$$

$$\frac{d}{dt}(x^2 + 8100) = \frac{d}{dt}(z^2)$$

$$2x \cdot \frac{dx}{dt} + 0 = 2z \cdot \frac{dz}{dt}$$

$$2(45)(-24) = 2(45\sqrt{5}) \cdot \frac{dz}{dt}$$

$$\frac{dz}{dt} = \frac{-24}{\sqrt{5}} \approx \boxed{-10.73 \text{ ft/s}}$$

b) Here, $y^2 + 90^2 = w^2$

so much is the same, except

$$\frac{dy}{dt} = +24 \text{ ft/sec}, \text{ so}$$

$$\frac{dw}{dt} = \frac{24}{\sqrt{5}} \approx \boxed{10.73 \text{ ft/sec}}$$