

## Logarithm:

How many times do we multiply a number by itself to get another number?

Consider:  $10 \times 10 \times 10 = 1000$

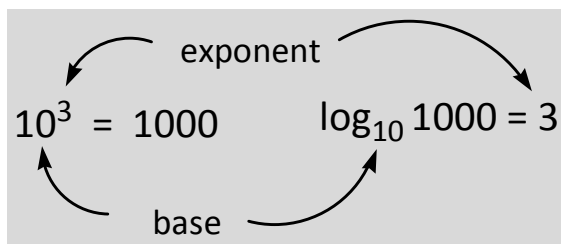
We multiply 10 by itself 3 times to get 1000- so the logarithm of 1000 is 3.

The number of times 10 is multiplied to get 1000 is 3:  $10^3 = 1000$

So the log expression is:  $\text{Log}_{10} 1000 = 3$

- The number we multiply is called the base, in this case 10 is the base
- The number of times we multiply is called the logarithm, in this case 3
- The number we generate is 1000

The relationship between exponent and log is this:



Summary:

Because multiplication and division are inverse operations,

- The number of times we multiply a number is the log of that number
- The number of times we divide a number is the negative log of that number

1000	$1 \times 10 \times 10 \times 10$	$\text{Log}_{10} 1000 =$	3	The base (subscripted <sub>10</sub> ) is not included in these expressions
100	$1 \times 10 \times 10$	$\text{Log } 100 =$	2	
10	$1 \times 10$	$\text{Log } 10 =$	1	
1	1	$\text{Log } 1 =$	0	
0.1	$1 / 10$	$\text{Log } 0.1 =$	-1	
0.01	$1 / 10 \times 10$	$\text{Log } 0.01 =$	-2	
0.001	$1 / 10 \times 10 \times 10$	$\text{Log } 0.001 =$	-3	

The common log is base 10 and often the base is not written in the log expression.

## Practice logs with your calculator.

### Example 1

Teach yourself the keystrokes needed to determine a log. Take the log of the values in Table 1 and compare your answer to the log values in the Table. For example:

$$\log 100 = 2 \qquad \log 1000 = 3 \quad \text{etc.}$$

### Example 2

Use your calculator and compare your answer to these values:

$$\log 0.001 = -3 \quad \log 0.005 = -2.3 \quad \log 3.4 \times 10^{-8} = -7.47$$

*Notice that for numbers less than 1, the log has a negative value.*

### Example 3 determining the negative log of a number

Use your calculator and compare your answer to these values:

$$-\log 1.0 \times 10^{-9} = 9 \qquad -\log 4.5 \times 10^{-5} = 4.35$$

### Inverse logs (also called antilogs)

Use your calculator and compare your answer to these values:

$$\text{invlog } -2.3 = 0.0050 \qquad \text{invlog } -11.6 = 2.5 \times 10^{-12}$$

## Application

To determine pH, calculate the  $-\log$  of the hydronium ion concentration  $[\text{H}_3\text{O}^+]$

$$[\text{H}_3\text{O}^+] = 5.75 \times 10^{-6}$$

$$\text{pH} = -\log (5.75 \times 10^{-6}) = 5.24$$

To determine  $[\text{H}_3\text{O}^+]$ , calculate the  $\text{invlog}$  of negative pH

$$\text{pH} = 10.2$$

$$[\text{H}_3\text{O}^+] = \text{invlog}(-\text{pH}) = \text{invlog}(-10.2) = 6.3 \times 10^{-11}$$