

Mathematics Review for Science Students



Math Department
Math Study Center
Grossmont College

1.0 - Exponents and Scientific Notation

1.1 - Exponential Notation

Exponential notation is a simple mathematical notation to represent repeated multiplication of the same number. Numbers in exponential notation are written in the form x^n , where x is the base and n is the exponent.

Examples:

1. $3^4 = 3 \times 3 \times 3 \times 3 = 81$

2. $10^3 = 10 \times 10 \times 10 = 1000$

1.2 - Some Rules of Exponents

1. $x^m \cdot x^n = x^{m+n}$

3. $x^{-n} = \frac{1}{x^n}$ for $x \neq 0$

2. $\frac{x^m}{x^n} = x^{m-n}$

4. $x^0 = 1$ for $x \neq 0$

Examples:

1. $10^3 \times 10^4 = 10^{3+4} = 10^7$

5. $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

2. $10^{-2} \times 10^5 = 10^{-2+5} = 10^3$

6. $10^{-4} = \frac{1}{10^4} = \frac{1}{10 \cdot 10 \cdot 10 \cdot 10} = \frac{1}{10000}$

3. $\frac{10^6}{10^3} = 10^{6-3} = 10^3$

7. $4^0 = 1$

4. $\frac{10^7}{10^{-4}} = 10^{7-(-4)} = 10^{7+4} = 10^{11}$

8. $10^0 = 1$

1.3 - Scientific Notation

Scientific notation is shorthand for writing very large and very small numbers. Proper scientific notation is written using powers of 10 and is in the form

$$N \times 10^m$$

where N is a number greater than or equal to 1 but less than 10 ($1 \leq N < 10$) and m is the exponent. For example, the speed of light, 1.86×10^5 miles per second is in scientific notation but 18.6×10^4 is not since N which is 18.6 is not between 1 and 10. In the example 1.86×10^5 , 10^5 means $10 \times 10 \times 10 \times 10 \times 10$ which is equal to 100000. This means that $1.86 \times 10^5 = 1.86 \times 100000 = 186,000$.

1.4 - Converting Numbers to Scientific Notation

Look at the number and count the number of positions you have to move the decimal point so that the resulting number is between 1 and 10. If you have to move the decimal point m places to the left, the exponent is positive m . If you move the decimal point m places to the right, the exponent is negative m .

Examples:

1. Write 186,000 in scientific notation.

- N must be 1.86 (between 1 and 10).
- Find the decimal point. If there is none, place the decimal point at the end of the number (186,000.)
- Move the decimal point to get 1.86 and count the number of places that you move it. Also note whether you are moving to the left or to the right.

186,000.
~~~~~

We moved 5 places to the left so the exponent must be 5 so  $186,000 = 1.86 \times 10^5$ .

2. Write -0.002341 in scientific notation.

- $N$  must be -2.341 (between 1 and 10).
- Find the decimal point.
- Move the decimal point to get -2.341 and count the number of places that you move it.

-0.00234  
~~~~

We moved 3 places to the left so exponent must be -3, so $-0.00234 = -2.34 \times 10^{-3}$.

1.4 Exercise Problems:

Write the following numbers in scientific notation:

- | | |
|------------------|---------------|
| 1. 31,536,000 | 3. -321.34 |
| 2. 0.00000006542 | 4. -0.0002376 |

1.5 - Entering Scientific Notation in Your Calculators

Most scientific calculators will have either an "EE" key or an "EXP" key that can be used for entering scientific notation. The TI-83 has an "EE" key.

To enter 123 or 1.23×10^2 , punch $\langle 1 \rangle \langle . \rangle \langle 2 \rangle \langle 3 \rangle \langle \text{EE} \rangle$ or $\langle \text{EXP} \rangle \langle 2 \rangle$. The calculator displays "1.23 2" (a space between the 1.23 and 2) or "1.23E2" on a graphing calculator such as the TI-83.

To enter 4.56×10^{-4} , punch $\langle 4 \rangle \langle . \rangle \langle 5 \rangle \langle 6 \rangle \langle \text{EE} \rangle$ or $\langle \text{EXP} \rangle \langle 4 \rangle \langle \pm \rangle$.

Warning: Do not confuse the "EE" key or an "EXP" key with the "y^x" or "x^y" key. Punching $\langle 1 \rangle \langle . \rangle \langle 2 \rangle \langle 3 \rangle \langle y^x \rangle$ or $\langle x^y \rangle \langle 2 \rangle$ means 1.23^2 not 1.23×10^2 .

1.6 - Multiplying and Dividing Numbers in Scientific Notation

Multiplying: Multiply the numbers “out front” (N parts) and then add the exponents.

$$\text{Example: } (2.4 \times 10^2)(3.0 \times 10^3) = (2.4)(3.0) \times (10^2)(10^3) = 7.2 \times 10^5$$

Dividing: Divide the numbers out front and then subtract the exponents.

$$\text{Example: } \frac{(4.8 \times 10^6)}{(1.2 \times 10^2)} = \frac{(4.8)}{(1.2)} \times \frac{(10^6)}{(10^2)} = 4.0 \times 10^{6-2} = 4.0 \times 10^4$$

If the product or the quotient of the two numbers out front doesn't fall between 1 and 10, you have to adjust it so that it is in proper scientific notation.

Examples:

$$1. (2.5 \times 10^2)(6.0 \times 10^3) = (2.5)(6.0) \times (10^2)(10^3) = 15 \times 10^5 = 1.5 \times 10^6$$

$$2. \frac{(1.0 \times 10^6)}{(50.0 \times 10^2)} = \frac{(1.0)}{(50.0)} \times \frac{(10^6)}{(10^2)} = 0.020 \times 10^{6-2} = 0.020 \times 10^4 = 2.0 \times 10^2$$

Notice what happened to the decimal point and exponent in each example. For every space you move the decimal to the left, you add one to the exponent. For every space you move the decimal to the right, you subtract one from the exponent (remember LARS – left/add; right/subtract).

1.6 Exercise Problems:

For each problem, perform the indicated operation and write your final answer in scientific notation:

$$1. (2.12 \times 10^5)(2.0 \times 10^3)$$

$$2. (6.0 \times 10^2)(6.0 \times 10^3)$$

$$3. 1,000,000 \times .002300$$

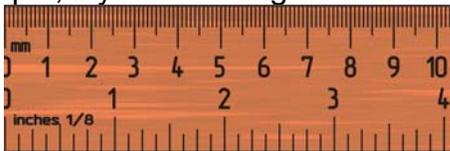
$$4. \frac{(3.0 \times 10^8)}{(2.0 \times 10^{-2})}$$

$$5. \frac{(-3.65 \times 10^{-3})}{(1.28 \times 10^6)}$$

2.0 – Rules for Significant Figures

2.1 - Significant Figures

Significant figures or significant digits are used to give us an idea of how precise a measurement is. For example, if you are using a ruler divided into millimeters,



you might try to read the scale to 0.1 mm. You may record a measurement of 8.2 mm (instead of 8.23 mm or 8.235 mm) and this is as precise as you can get since you can only measure to the nearest 0.1 mm. The measurement 8.2 mm has two significant figures.

2.2 - Rules for Significant Figures

Any digit that is actually measured or estimated will be considered significant. Placeholders, or digits that have not been measured or estimated, are not considered significant. The rules for determining the significance of a digit are as follows:

1. If the number has a decimal point:

Reading from left to right, the first non-zero digit and all the remaining digits are significant figures.

Examples:

- | | |
|-----------|---|
| 0.0023056 | has 5 significant figures (the first 3 are placeholders) |
| 720.2150 | has 7 significant figures (the last 0 counts) |
| 0.010 | has 2 significant figures (the 1 st two zeros don't count) |

2. If the number has no decimal point:

Only the first and last non-zero digit and all digits in between those two are significant.

Examples:

- | | |
|-------|---|
| 20000 | has 1 significant figure (the trailing zeroes are placeholders) |
| 30200 | has 3 significant figures |

2.2 Exercise Problems:

For each number, determine the number of significant figures:

1. 12.5

3. 0.001250

5. 10250

2. 12.50

4. 125000

6. 102.005

2.3 - Significant Figures in Multiplication and Division

In multiplication and division, the result should be rounded off so that it has the same number of significant figures as in the measurement with the fewest number of significant figures.

Example:

| | |
|--------------|---|
| 3.126 | (has four significant figures) |
| x <u>3.1</u> | (has two significant figures) |
| 9.6906 | (but must be rounded to 9.7 since 3.1 has the fewest |
| ↓ | number of significant figures which is two) |
| 9.7 | (final answer with two significant figures) |

2.4 - Significant Figures in Addition and Subtraction

In addition and subtraction, the result is rounded off so that it has the same number of decimal places as the measurement having the fewest decimal places.

Example:

| | |
|-----------------|---|
| 123.2 | (has one decimal place) |
| + <u>34.521</u> | (has three decimal places) |
| 157.721 | (but must be rounded to 157.7 since 123.2 has the fewest |
| ↓ | decimal places which is 1 decimal place) |
| 157.7 | (final answer with one decimal place) |

2.3 and 2.4 Exercise Problems:

For each problem, perform the indicated operation and write your answer using the correct number of significant figures.

1. $17.761 + 2.90 + 1126.4$

2. $2319.25 - 532.613$

3. $1054.630 + 327.0362 + 5.64$

4. $125 - 0.23 + 4.109$

5. 2.02×2.5

6. $500.0 / 4.3021$

7. 0.0052×673

8. $(3.3)^3$

9. $0.553 \times (50.0 - 22.3)$

10. 435×4.00

3.0 – Units and Unit Conversions

3.1 – Units

For a measurement to make sense, it should be expressed with the appropriate units. If you state that the length of an object is 10, it won't be surprising for your instructor to say, "*Ten What???*" since it could be 10 meters or 10 yards.

Below are some examples of the quantities you might measure and the units you might use.

Length: millimeter(mm), centimeter(cm), inch(in), foot(ft), meter(m), kilometer (km)

Mass: milligram (mg), gram (g), kilogram (kg)

Time: seconds (s): minute (min), hour (hr)

Temperature: Fahrenheit (°F), Kelvin (K), Celsius (°C)

3.2 - Unit Conversions

You may need to change between different units such as changing from inches to feet or changing from g/cm^3 in units of kg/m^3 . When converting from one unit to another, we use Dimensional Analysis, also known as the Unit Factor Method. This uses the idea that any number or expression can be multiplied by a factor of one without changing its value. Thus, to convert units, you must know the necessary conversion factor(s).

Examples:

1. Convert 2.5 inches to centimeters.

Step 1: Find the conversion factor.

$$1 \text{ inch} = 2.54 \text{ centimeters (cm)} \text{ or } \frac{1 \text{ inch}}{2.54 \text{ cm}} \text{ or } \frac{2.54 \text{ cm}}{1 \text{ inch}}$$

Step 2: Find what units you want the final answer in and find the right form of the conversion factor that you will use.

Since we the final answer to be in cm, use the conversion factor that has cm on top: $\frac{2.54 \text{ cm}}{1 \text{ inch}}$

Step 3: Setup each problem by writing down what you need to find using a question mark followed by the units. Set this equal to the information that you are given and multiply the right hand side of the equation with the conversion factor. Cancel the units that give a factor of one (in short, cancel the same units that appear between the top and bottom).

$$? \text{ cm} = 2.5 \text{ inches} \times \frac{2.54 \text{ cm}}{1 \text{ inch}} = 6.4 \text{ cm}$$

2. What is the density of mercury (13.6 g/cm^3) in units of kg/m^3 ?

Step 1: Find the conversion factors you will use.

$$\text{To go from g to kg: } \frac{1 \text{ kg}}{1000 \text{ g}} \text{ or } \frac{1000 \text{ g}}{1 \text{ kg}}$$

$$\text{To go from cm to m: } \frac{1 \text{ m}}{100 \text{ cm}} \text{ or } \frac{100 \text{ cm}}{1 \text{ m}}$$

Step 2: Find what units you want the final answer in and find the right form of the conversion factor that you will use.

Since we want the final answer to be in kg/m^3 , use the conversion factors that has kg on top and m on the bottom: $\frac{1 \text{ kg}}{1000 \text{ g}}$, $\frac{100 \text{ cm}}{1 \text{ m}}$

Step 3: Setup each problem by writing down what you need to find using a question mark followed by the units. Set this equal to the information that you are given and multiply the right hand side of the equation with the conversion factor. Cancel the units that give a factor of one (in short, cancel the same units that appear between the top and bottom).

$$\begin{aligned} ? \frac{\text{kg}}{\text{m}^3} &= 13.6 \frac{\text{g}}{\text{cm}^3} \times \frac{(100 \text{ cm})^3}{1 \text{ m}^3} \times \frac{1 \text{ kg}}{1000 \text{ g}} \\ &= 13.6 \frac{\cancel{\text{g}}}{\cancel{\text{cm}^3}} \times \frac{1000000 \cancel{\text{cm}^3}}{1 \text{ m}^3} \times \frac{1 \text{ kg}}{1000 \cancel{\text{g}}} \\ &= 13600 \frac{\text{kg}}{\text{m}^3} = 1.36 \times 10^4 \text{ kg/m}^3 \end{aligned}$$

4.0 - Algebra Review

4.1 - Solving for a Specific Variable in an Algebraic Equation

You will be working with a variety of formulas in your class. One example of such formulas is the Density formula: Density = Mass/Volume or $D = \frac{M}{V}$.

You may be asked to find the value of any one of these variables so you have to be comfortable in solving for a specific variable in an equation. In the above example, you may be asked to solve for either D or V . Remember that the goal is to keep the variable in question on one side of the equation and move the rest to the other side. You can do this by performing the appropriate operation or a series of operations like adding, subtracting, multiplying, dividing, raising to a power, or taking the root on both sides of the equation.

Example: Given the formula $D = \frac{M}{V}$, solve for V .

- Eliminate any fractions first by multiplying both sides of the equal sign by V .

$$V \cdot D = \frac{M}{V} \cdot V \implies VD = M$$

- Isolate V by dividing both sides of the equal sign by D (think of the reverse operations – multiplication/division, addition/subtraction, squaring/square root).

$$\frac{VD}{D} = \frac{M}{D} \implies V = \frac{M}{D}$$

4.1 Exercise Problems:

1. Given the formula $v = \lambda f$, where v = speed, λ = wavelength, f = frequency

- Solve for λ
- Solve for f

2. Given the formula $P.E. = \frac{1}{2} kx^2$, where $P.E.$ = Potential Energy, k = force

constant of spring, x = distance the spring is stretched.

- Solve for k
- Solve for x

4.2 – Change in Quantity

In mathematics, the Greek letter Δ (delta) is used to denote a change in quantity or the difference between 2 quantities.

Examples:

1. Slope of a line: $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}$

2. Change in time: $\Delta t = t_2 - t_1$

3. Temperature change: $\Delta T = T_2 - T_1$

5.0 - Graphing Data

5.1 - Graphing Guidelines

Here are some guidelines that you can use when preparing graphs for your class. Make sure that you ask your respective instructors for any class specific requirements.

1. **Give a title.** Include a title that describes the purpose of the graph and what the data represent (e.g. "Temperature versus Time for the Reaction of X with Y").
2. **Label each axis.** Indicate the quantity and units of each variable (e.g., Mass (g)). The independent variable (variable you are manipulating) is labeled on the x-axis while the dependent variable (variable that changes in response to the independent variable) is labeled on the y-axis.
3. **Choose an appropriate range.** Note the minimum and maximum values of each variable to determine the range of each axis.
4. **Choose an appropriate scale that maximizes your graph area.** Use convenient increments which are easy to read such as 1, 2, 5, or a multiple of these numbers such as 0.1, 10, 20, 100, etc). For example, if your data ranges from 1 cm to 34 cm, you may want to represent each division to be 5 cm but if your data ranges from 1 to 195, you may want to divide it into 20 cm each. Ensure that you maintain a uniform scale along each axis (the x-axis and the y-axis do not need to have the same scale).
5. **Plot your points clearly and accurately.** Use an X or a dot with a small circle around it so the point can still be seen after you draw a line or a curve.
6. **Draw a best-fit line or curve.** Your graphs will not all fall exactly on a straight line or a curve so draw one that best represents the general tendency of the data points. If you have to draw a line of best fit and the plot is a straight line, use a ruler. If it is a curve, draw a smooth curve by freehand. Try not to simply connect all the dots!

5.1 Exercise Problem:

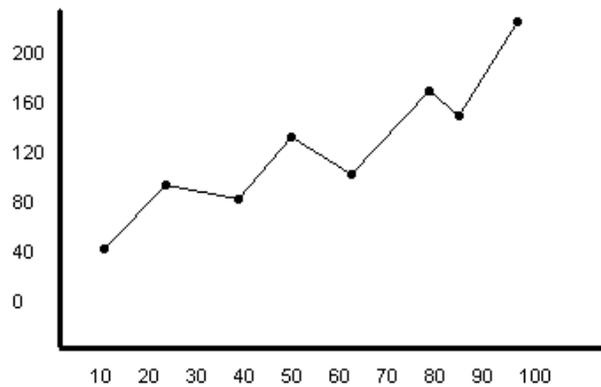
Six students provided the following data about the lengths of time they studied for a math exam and the grades they received on the exam. Draw a graph that represents the given set of data.

| Time studied (minutes) | Grade received (percent) |
|------------------------|--------------------------|
| 20 | 40 |
| 40 | 46 |
| 50 | 70 |
| 60 | 75 |
| 80 | 92 |
| 100 | 95 |

5.2 – Examples of Good and Bad Graphs

(from <http://misterguch.brinkster.net/graph.html>)

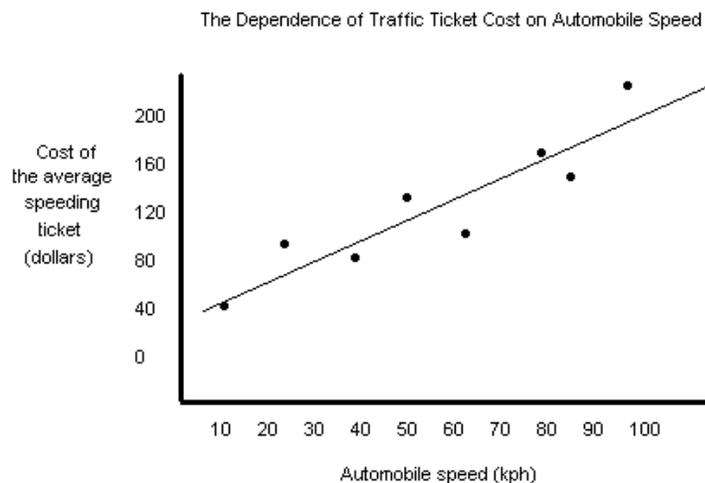
A bad graph!



Let's see what's wrong with this graph:

- There's no title. What's it a graph of? Who knows?
- There are no labels on the x or y axis. What are those numbers? Who knows?
- There are no units on the x or y axis. Is this a graph of speed in miles per hour or a graph of temperature in Kelvins? Who can tell?
- Somebody played "connect the dots". This should be a nice straight line which goes through the points or a curve that tends to follow them.

A good graph!



Doesn't the clarity and beauty of this graph just make you want to cry? Well, maybe that's overstating it a little bit, but it sure does make more sense than the first one, doesn't it? I'm starting to mist up right now.* *No, I'm not.

Answers to Exercise Problems

1.4 – Converting Numbers to Scientific Notation

1. 3.1536×10^7
2. 6.542×10^{-8}
3. -3.2134×10^2
4. -2.376×10^{-4}

1.6 – Multiplying and Dividing Numbers in Scientific Notation

1. 4.2×10^8
2. 3.6×10^6
3. 2.300×10^3
4. 1.5×10^{10}
5. -2.85×10^{-9}

2.2 – Determining the Number of Significant Figures

1. 3 significant figures
2. 4 significant figures
3. 4 significant figures
4. 3 significant figures
5. 4 significant figures
6. 6 significant figures

2.3 and 2.4 – Adding/Subtracting/Multiplying/Dividing Numbers in Significant Figures

1. 1147.1

2. 1786.64
3. 1387.31
4. 129
5. 5.1
6. 116.2
7. 3.5
8. 36
9. 15.3
10. 1740

3.2 – Unit Conversion

1. .0400 liters or 4.00×10^{-2} liters
2. 9.5×10^7 seconds
3. 9.1×10^{-25} milligrams
4. 22.2 m/s

4.1 – Solving for a Specific Variable in an Equation

1.

a. $\lambda = \frac{v}{f}$

b. $f = \frac{v}{\lambda}$

2.

a. $k = \frac{2(P.E.)}{x^2}$

b. $x = \sqrt{\frac{2(P.E.)}{k}}$

5.1 – Graphing

