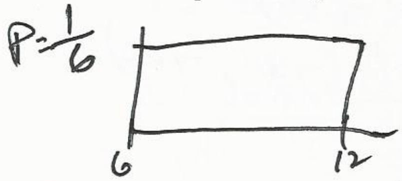
Practice Exam #3: Chapters 6 - 7 NAME Answer Key
Math 160

Show all work to receive full credit. You may use a calculator. CHECK YOUR WORK!!!!

THERE ARE 4 PAGES IN THIS EXAM

Write down all formulas or calculator commands used to receive credit!!!!!!!!

1. Assume that the weight loss for the first month of a diet program varies between 6 pounds and 12 pounds and is spread evenly over the range of possibilities, so that there is a uniform distribution. Find the probability of more than 10 pounds lost.



$P(x > 10) = (12-10)\left(\frac{1}{6}\right) = \frac{2}{6} = \frac{1}{3}$

$= \boxed{0.333}$

2. Find the margin of error for the following: 98% confidence interval, $n = 91$, $\bar{x} = 53$, $s = 12.5$.

$E = t_{\alpha/2} \frac{s}{\sqrt{n}}$ $\alpha = 0.02$ $\alpha/2 = 0.01$ $E = \frac{2.368\,(12.5)}{\sqrt{91}} = 3.102923$

$df = 90$

$t_{0.01,90} = 2.368$ $\boxed{E = 3.1}$ 1 DECIMAL BECAUSE OF S.

3. To perform a z-confidence interval, what three simple conditions must be met?

1.) σ KNOWN

2.) $n > 30$
OR
3.) POPULATION NORMALLY DISTRIBUTED

4. Students living away from home for the first time often face many stresses and therefore change their dietary and exercise habits. It is reported that freshmen living away from home for the first time and eating a typical college diet usually gain weight with a mean of 15 pounds and a standard deviation of 10 pounds. Assume the change in student weight is normally distributed.

A. Find the approximate probability that a given student will not gain weight. $P(X \le 0)$

$\mu = 15 \quad \sigma = 10$ LBS

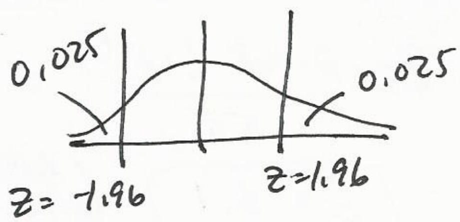
$$Z = \frac{X-\mu}{\sigma} = \frac{0-15}{10} = -1.50$$

$$P(X \le 0) = P(Z < -1.50)$$

$$= \text{NORMALCDF}(-1000, -1.50)$$

$$= \boxed{0.0668}$$

B. In what range do the middle 95% of all weight changes lie?



$X_R = 15 + 1.96(10)$
$= 34.6$

MIDDLE 95%
-4.6 LBS TO 34.6 LBS

$X = \mu + Z\sigma$
$X_L = 15 - 1.96(10)$
$= -4.6$

C. For 25 randomly selected students, find the probability that the change in student weight will have a mean greater than 10 pounds.

$$P(\bar{X} > 10) = P(Z > -2.50)$$

$$Z = \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} = \frac{10-15}{10/\sqrt{25}} = \frac{-5}{10/5} = \frac{-5}{2} = -2.50$$

$$P(Z > -2.50) = \text{NORMALCDF}(-2.50, 1000)$$

$$= \boxed{0.9938}$$

5. The daily intakes of milk (in ounces) for ten randomly selected people were:

17.0 12.1 18.6 16.2 17.8 24.3 26.7 10.3 27.0 30.3 $\Rightarrow L_1$

1 VARSTATS $L_1 \Rightarrow$

$\bar{x} = 20.03$

$s = 6.709868$

D. Find a 99% confidence interval for the population mean μ. Assume the population has a standard normal distribution.

$\alpha = 0.01 \quad df = 9$

$\alpha/2 = 0.005$

$t_{0.005,9} = 3.250$

$E = t_{\alpha/2}\frac{s}{\sqrt{n}} = 3.250\left(\frac{6.709868}{\sqrt{10}}\right)$

$= 6.89600$

$\bar{x} - E < \mu < \bar{x} + E$

$20.03 - 6.896 < \mu < 20.03 + 6.896$

$13.134 < \mu < 26.926$

$\boxed{13.13 < \mu < 26.93 \text{ oz}}$

E. Find a 99% confidence interval for the population standard deviation σ.

$\sqrt{\frac{(n-1)s^2}{\chi^2_R}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi^2_L}}$

$\chi^2_R = \chi^2_{0.005,9} = 23.589$

$\chi^2_L = \chi^2_{0.995,9} = 1.735$

$\sqrt{\frac{9(6.709868)^2}{23.589}} < \sigma < \sqrt{\frac{9(6.709868)^2}{1.735}}$

$4.1446 < \sigma < 15.2822$

$\boxed{4.14 < \sigma < 15.28 \text{ oz}}$

6. Do one of the following as appropriate: a) Find the critical value $z_{\alpha/2}$, b) find the critical value $t_{\alpha/2}$, c) state neither the normal nor the t distribution apply (state why).

95%, $n = 11$, σ is known; population appears to be very skewed.

NEITHER σ KNOWN $\Rightarrow$ IMPLIES Z INTERVAL BUT, $n < 30$ AND POP. NOT NORMALLY DISTRIBUTED.

7. Find the appropriate minimum sample size. You want to be 99% confident that the sample standard deviation s is within 5% of the population standard deviation.

FROM CHART
BOTTOM RIGHT
TABLE 7-2

$\boxed{n = 1335}$

8. In a survey of 4800 TV viewers, 50% said they watch network news programs. Find the margin of error for the 95% confidence interval used to estimate the population proportion.

$\hat{p} = 0.5$

$E = z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$

$= 1.96\sqrt{\frac{(0.5)(0.5)}{4800}} = 0.0141451$

$= \boxed{0.0141}$

9. Find the minimum sample size you should use to assure that your estimate of *p*, the population proportion, will be within a margin of error of 0.022 for a 93% confidence level where *p* and *q* are unknown.

$\hat{p} = \hat{q} = 0.5 \qquad Z_{0.035} = 1.81$

$$n = \frac{(Z_{\alpha/2})^2 \hat{p}\hat{q}}{E^2}$$

$$= \frac{(1.81)^2(0.5)(0.5)}{(0.022)^2} = 1692.200413 \Rightarrow \boxed{1693}$$

10. A final exam in Math 160 has a mean of 74 with a standard deviation of 8.5. If 24 students are randomly selected, find the probability that their mean of their test scores is less than 70.

$\mu = 74$

$\sigma = 8.5$

$n = 24$

$P(\bar{X} < 70)$

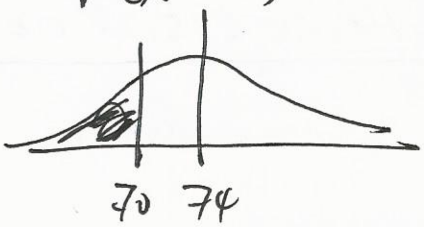
$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} = \frac{70 - 74}{8.5/\sqrt{24}}$$

$$= -2.3054 = -2.31$$

$$P(\bar{X} < 70) = P(Z < -2.31)$$

$$= \text{NORMALCDF}(-1000, -2.31)$$

$$= \boxed{0.0104}$$



11. Use the given confidence interval (0.868, 0.890) to find the point estimate $\hat{p}$ and the margin of error E.

$$\hat{p} = \frac{MAX + MIN}{2} \qquad E = \frac{MAX - MIN}{2}$$

Point Estimate: 0.879

Margin of Error: 0.011

$$= \frac{0.890 + 0.868}{2} \approx 0.879$$

$$= \frac{0.890 - 0.868}{2} = 0.011$$

12. When 298 college students are randomly selected and surveyed, it is found that 111 own a car. Find a 99% confidence interval for the true proportion of all college students who own a car. (Round your answers to three decimal.)

$X = 111$

$n = 298$

$\alpha = 0.01$

$\alpha/2 = 0.005$

$z_{0.005} = 2.575$

$\hat{p} = \frac{111}{298} = 0.372483$

$\hat{q} = \frac{187}{111} = 0.627517$

$E = z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$

$= 2.575\sqrt{\frac{(0.372483)(0.627517)}{298}}$

$= 0.0721166$

$= 0.072117$

$\hat{p} - E < p < \hat{p} + E$

$0.372483 - 0.072117 < p <$

$0.372483 + 0.072117$

$0.300366 < p < 0.4446$

$\boxed{0.300 < p < 0.445}$