PRACTICE EXAM: Chapters 6 - 7 NAME ____________________

Math 160

Michael Orr

100 points. Show all work to receive full credit. You may use a calculator. CHECK YOUR WORK!!!!

THERE ARE 4 PAGES IN THIS EXAM

Write down all formulas or calculator commands used to receive credit!!!!!!!!

1. (8 pts) Based on census records, a certain city is thought to have a demographic makeup that is 22% Hispanic. If a random sample of 50 people is chosen, what is the probability that the sample contains more than 15 Hispanics. Use the normal approximation to the binomial distribution. Round to the nearest ten thousandths.

$P(X>15) = P(X>15.5)$

$n = 50$

$p = 0.22$

$\mu = np = 11$

$\sigma = \sqrt{npq} = \sqrt{50(0.22)(0.78)} = 2.929164$

$z = \dfrac{x-\mu}{\sigma} = \dfrac{15.5-11}{2.929164}$

$z = 1.5363 = 1.54$

$P(X>15) = P(X>15.5) = P(z>1.54)$

$=$ NORMALCDF $(1.54, 1000)$

$= \boxed{0.0618}$

2. (16 pts total) The football coach randomly selected twelve players and timed how long each player took to complete a certain drill. The times (in minutes) were:

$n = 12$

6 10 7 9 12 8 10 9 7 5 11 8 $\rightarrow L_1$ 1VARSTATS L_1 $\bar{x} = 8.5$ $s = 2.067058$

$df = 11$

A. Find a 98% confidence interval for the population mean μ. Assume the population has a standard normal distribution.

$\alpha = 0.02$

$\alpha/2 = 0.01$

$t_{0.01, 11} = 2.718$

$E = t_{\alpha/2}\dfrac{s}{\sqrt{n}} = \dfrac{2.718(2.067058)}{\sqrt{12}} = 1.62185$

$\bar{x} - E < \mu < \bar{x} + E$ MIN

$8.5 - 1.62185 < \mu < 8.5 + 1.62185$

$6.87815 < \mu < 10.12185$

$\boxed{6.9 < \mu < 10.1 \text{ MIN}}$

B. Find a 98% confidence interval for the population standard deviation σ.

$$\sqrt{\frac{(n-1)s^2}{\chi^2_R}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi^2_L}}$$

$\chi^2_R = \chi^2_{0.01, 11} = 24.725$

$\chi^2_L = \chi^2_{0.99, 11} = 3.053$

$$\sqrt{\frac{11(2.067058)^2}{24.725}} < \sigma < \sqrt{\frac{11(2.067058)^2}{3.053}}$$

$1.378735 < \sigma < 3.923610$

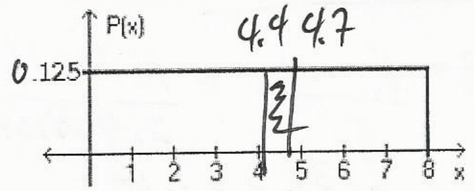
$\boxed{1.4 < \sigma < 3.9 \text{ MIN}}$

3. (4 pts) Do one of the following as appropriate: a) Find the critical value $z_{\alpha/2}$, b) find the critical value $t_{\alpha/2}$, c) state neither the normal nor the t distribution apply (state why).

90%; $n=9$; $\sigma=4.2$; population appears to be very skewed.

NEITHER. EVEN THOUGH σ IS KNOWN, $n < 30$ AND POPULATION IS NOT NORMALLY DISTRIBUTED

4. (8 pts) Using the following uniform density curve, what is the probability that the random variable has a value between 4.4 and 4.7? (Use three decimal places)



$$P(4.4 < x < 4.7) = (4.7-4.4)(0.125)$$
$$= 0.3(0.125)$$
$$= 0.0375 = \underline{0.038}$$

5. (8 pts) Find the margin of error for the following: 80% confidence interval, $n=51$, $\bar{x}=129$, $s=274$.

$$E = t_{\alpha/2}\frac{s}{\sqrt{n}} \qquad E = \frac{1.299\,(274)}{\sqrt{51}}$$

$df = n-1 = 50$ $\qquad = 49.8396 \quad \Rightarrow \boxed{50}$

$t_{0.10,\,50} = 1.299$

6. (8 pts) Suppose you are interested in estimating the percentage of all California high school students who passed the high school exit exam on the first try. If the goal is to estimate the percentage with 95% confidence and a margin of error of 7%, how many current California high school students' records should be sampled?

$$n = \frac{(z_{\alpha/2})^2\,\hat{p}\hat{q}}{E^2} \qquad \hat{p} = \hat{q} = 0.5 \qquad E = 0.07$$

$$= \frac{(1.96)^2(0.5)(0.5)}{(0.07)^2} = 196 \qquad z_{\alpha/2} = 1.96$$

$\boxed{\text{SAMPLE SIZE } n = 196}$

$n=44$

7. (8 pts) A final exam in Math 160 has a mean of 72 with a standard deviation of 12.5. If 44 students are randomly selected, find the probability that their mean of their test scores is greater than 75. (Round your answers to three decimal places.)

$$z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} = \frac{75-72}{12.5/\sqrt{44}} = 1.5919799 = 1.59$$

$$P(\bar{x} > 75) = P(z > 1.59) = P(z < -1.59)$$
$$= \text{normalcdf}(1.59, 1000) = 0.055917$$
$$= \boxed{0.056}$$

8. (8 pts) A survey of 300 union members in California reveals that 112 favor the Meg Whittman for governor. Find the margin of error for the 93% confidence interval used to estimate the true population proportion of all California union members who favor the Republican candidate. Round to the nearest thousandths.

$E = Z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$

$\hat{p} = \frac{x}{n} = \frac{112}{300} = 0.373333$

$\hat{q} = \frac{188}{300} = 0.626667$

$\alpha = 0.07$

$\alpha/2 = 0.035$

$Z_{0.035} = 1.81$

$E = 1.81\sqrt{\frac{(0.373333)(0.626667)}{300}} = 0.050586$

$\boxed{E = 0.051}$

9. (8 pts) A random sample of light bulbs is to be taken so that the owner of the company can estimate the average life of his light bulbs. How many light bulbs should he sample so that he can estimate the mean life with 99% confidence and a margin of error of 5 hours? Assume that it is known that the standard deviation for light bulb lifetimes is 10 hours.

$n = \left(\frac{Z_{\alpha/2}\,\sigma}{E}\right)^2 = \left[\frac{2.575(10)}{5}\right]^2 = 26.5225$

$= \boxed{27 \text{ LIGHT BULBS}}$

10. (10 pts) The probability that a radish seed will germinate is 0.7. Estimate the probability that of 140 randomly selected seeds, exactly 100 will germinate. Use the normal approximation to the binomial distribution. Round to 4 decimal places.

$P(X=100) = P(99.5 < x < 100.5)$

$n = 140$

$p = 0.7$

$\mu = np = 140(0.7) = 98$

$\sigma = \sqrt{npq} = \sqrt{140(0.7)(0.3)} = 5.422177$

$X = 99.5 \quad Z = \frac{X-\mu}{\sigma} = \frac{99.5-98}{5.422177} = 0.2766 = 0.28$

$X = 100.5 \quad Z = \frac{100.5-98}{5.422177} = 0.4611 = 0.46$

$= P(0.28 < Z < 0.46)$

$=$ NORMALCDF $(0.28, 0.46) = 0.066981$

$= \boxed{0.0670}$

11. (6 pts) A confidence interval was calculated to estimate the average number of hours per day that females in the U.S. watch TV. This yielded the following confidence interval: (2.88, 3.26). Which of the following is the true statement?

$\bar{X} = \frac{\text{MAX}+\text{MIN}}{2}$

$E = \frac{\text{MAX}-\text{MIN}}{2}$

- **A.** The sample mean was $\bar{X} = 3.07$ and the margin of error was 0.38 hours.
- **B.** The sample mean was $\bar{X} = 3.07$ and the margin of error was 0.19 hours.
- **C.** The population mean was $\mu = 3.07$ and the margin of error was 0.19 hours.
- **D.** The population mean is $\mu = 3.07$ and the margin of error was 0.19 hours.
- **E.** We don't know the population mean, but we do know that $\hat{p} = 3.07$.
- **F.** None of the above statements is true.

12. (8 pts) A Gallup Youth Survey in 2001 found that of 501 randomly selected teenagers in the US, 271 of them said that they got along "very well" with their parents. Find a 99% confidence interval for the true proportion of all teenagers who get along "very well" with their parents. (Round your answers to three decimal.)

$\hat{p} = \frac{271}{501} = 0.540918$

$\hat{q} = 230/501 = 0.459082$

$E = z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$

$E = 2.575\sqrt{\frac{0.540918(0.459082)}{501}}$

$= 0.057328$

$\hat{p} - E < p < \hat{p} + E$

$0.540918 - 0.057328 < p <$
$0.540918 + 0.057328$

$0.48359 < p < 0.598246$

$\boxed{0.484 < p < 0.598}$

BONUS (10 points)

(5 pts) A company purchases shipments of machine components and uses this acceptance sampling plan: Randomly select and test 26 components and accept the whole batch if there are fewer than 3 defectives. If a particular shipment of thousands of components actually has a 3% rate of defects, what is the probability that this whole shipment will be accepted? Use four decimal places in your answer.

(5 pts) A bag contains 6 cherry, 5 orange, and 4 lemon candies. You reach in and take 3 pieces of candy at random. Find the probability that you have one candy of each flavor. Use 3 decimal places.