Chapter 12

Probability

WHAT YOU WILL LEARN

- Empirical probability and theoretical probability
- Compound probability, conditional probability, and binomial probability
- Odds against an event and odds in favor of an event
- Expected value
- Tree diagrams

WHAT YOU WILL LEARN

- Mutually exclusive events and independent events
- The counting principle, permutations, and combinations

Section 1

The Nature of Probability

Definitions

- An **experiment** is a controlled operation that yields a set of results.
- The possible results of an experiment are called its **outcomes**.
- An **event** is a subcollection of the outcomes of an experiment.

Definitions (continued)

- **Empirical probability** is the relative frequency of occurrence of an event and is determined by actual observations of an experiment.
- **Theoretical probability** is determined through a study of the possible *outcomes* that can occur for the given experiment.

Empirical Probability

$$P(E) = \frac{\text{number of times event } E \text{ has occurred}}{\text{total number of times the experiment has been performed}}$$

- Example: In 100 tosses of a fair die, 19 landed showing a 3. Find the empirical probability of the die landing showing a 3.
- Let E be the event of the die landing showing a 3.

$$P(E) = \frac{19}{100} = 0.19$$

The Law of Large Numbers

- The **law of large numbers** states that probability statements apply in practice to a large number of trials, not to a single trial. It is the relative frequency over the long run that is accurately predictable, not individual events or precise totals.

Section 2

Theoretical Probability

Equally likely outcomes

- If each outcome of an experiment has the same chance of occurring as any other outcome, they are said to be **equally likely outcomes**.
- For equally likely outcomes, the probability of Event E may be calculated with the following formula.

$$P(E) = \frac{\text{number of outcomes favorable to } E}{\text{total number of possible outcomes}}$$

Example

- A die is rolled. Find the probability of rolling
- a) a 2.
- b) an odd number.
- c) a number less than 4.
- d) an 8.
- e) a number less than 9.

Solutions: There are six equally likely outcomes: 1, 2, 3, 4, 5, and 6.

- a)

$$P(2) = \frac{\text{number of outcomes that will result in a 2}}{\text{total number of possible outcomes}} = \frac{1}{6}$$

- b) There are three ways an odd number can occur: 1, 3 or 5.

$$P(\text{odd}) = \frac{3}{6} = \frac{1}{2}$$

- c) Three numbers are less than 4.

$$P(\text{number less than 4}) = \frac{3}{6} = \frac{1}{2}$$

Solutions: There are six equally likely outcomes: 1, 2, 3, 4, 5, and 6 (continued)

- d) There are no outcomes that will result in an 8.

$$P(\text{number greater than } 8) = \frac{0}{6} = 0$$

- e) All outcomes are less than 9. The event must occur and the probability is 1.

Important Facts

- The probability of an event that cannot occur is 0.
- The probability of an event that must occur is 1.
- Every probability is a number between 0 and 1 inclusive; that is, $0 \le P(E) \le 1$.
- The sum of the probabilities of all possible outcomes of an experiment is 1.

Example

- A standard deck of cards is well shuffled. Find the probability that the card is selected.
  a) a 10.
  b) not a 10.
  c) a heart.
  d) an ace, 2, or 3.
  e) diamond and spade.
  f) a card greater than 4 *and* less than 7.

Example (continued)

a) a 10

There are four 10's in a deck of 52 cards.

$$P(10) = \frac{4}{52} = \frac{1}{13}$$

b) not a 10

$$P(\text{not a } 10) = 1 - P(10)$$
$$= 1 - \frac{1}{13}$$
$$= \frac{12}{13}$$

Example continued

c) a heart

There are 13 hearts in the deck.

$$P(\text{heart}) = \frac{13}{52} = \frac{1}{4}$$

d) an ace, 2 or 3

There are 4 aces, 4 twos and 4 threes, or a total of 12 cards.

$$P(\text{A, 2, or 3}) = \frac{12}{52} = \frac{3}{13}$$

Example continued

d) diamond and spade

The word *and* means both events must occur. This is not possible.

$$P(\text{diamond and spade}) = \frac{0}{52} = 0$$

e) a card greater than 4 and less than 7

The cards greater than 4 and less than 7 are 5's and 6's (or a total of 8 cards).

$$P(> 4 \text{ and } < 7) = P(5 \text{ or } 6) = \frac{8}{52} = \frac{2}{13}$$