Section 12.5

Tree Diagrams

Counting Principle

- If a first experiment can be performed in *M* distinct ways and a second experiment can be performed in *N* distinct ways, then the two experiments in that specific order can be performed in $M \cdot N$ distinct ways.

Definitions

- Sample space: A list of all possible outcomes of an experiment.
- Sample point: Each individual outcome in the sample space.
- Tree diagrams are helpful in determining sample spaces.

Example

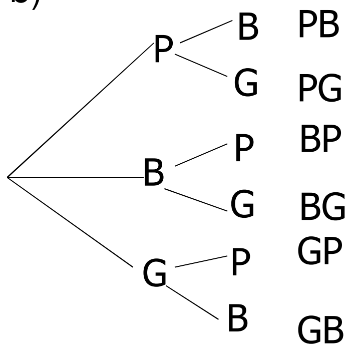
- Two balls are to be selected *without replacement* from a bag that contains one purple, one blue, and one green ball.

a) Use the counting principle to determine the number of points in the sample space.
b) Construct a tree diagram and list the sample space.
c) Find the probability that one blue ball is selected.
d) Find the probability that a purple ball followed by a green ball is selected.

Solutions

a) 3 • 2 = 6 ways

b)



c) $P(\text{blue}) = \frac{4}{6} = \frac{2}{3}$

d) $P(\text{Purple,Green})$

$P(\text{P,G}) = \frac{1}{6}$

P(event happening at least once)

$$P\begin{pmatrix}\text{event happening}\\ \text{at least once}\end{pmatrix} = 1 - P\begin{pmatrix}\text{event does}\\ \text{not happen}\end{pmatrix}$$

Examples

- At a homeowners' association meeting, a board member can vote yes, no, or abstain a motion. There are three motions on which a board member must vote.

A. Determine the number of points in the sample space.
B. Construct a tree diagram and determine the sample space.
C. Determine the probability that a board member votes no, yes, no in that order.
D. Determine the probability that a board member votes yes on exactly two of the motions.
E. Determine the probability that a board member votes yes on at least one of the motions.

Examples

- An individual can be classified as male or female with red, brown, black, or blond hair and with brown, green, or blues eyes.

A. How many different classifications are possible?
B. Construct a tree diagram to determine the sample space.
C. If each outcome is equally likely, determine the probability that the individual will be a male with black hair and blue eyes.
D. Determine the probability that the individual will be a female with blond hair.

Section 12.6

Or and *And* Problems

Or Problems

- $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$
- Example: Each of the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10 is written on a separate piece of paper. The 10 pieces of paper are then placed in a bowl and one is randomly selected. Find the probability that the piece of paper selected contains an even number or a number greater than 5.

Solution

- $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$$P\begin{pmatrix}\text{even or} \\ \text{greater than 5}\end{pmatrix} =$$

$$P(\text{even}) + P(\text{greater than 5}) - P\begin{pmatrix}\text{even and} \\ \text{greater than 5}\end{pmatrix}$$

$$= \frac{5}{10} + \frac{5}{10} - \frac{3}{10} = \frac{7}{10}$$

- Thus, the probability of selecting an even number or a number greater than 5 is 7/10.

Example

- Each of the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10 is written on a separate piece of paper. The 10 pieces of paper are then placed in a bowl and one is randomly selected. Find the probability that the piece of paper selected contains a number less than 3 or a number greater than 7.

Solution

$$P(\text{less than } 3) = \frac{2}{10}$$

$$P(\text{greater than } 7) = \frac{3}{10}$$

There are no numbers that are *both* less than 3 and greater than 7. Therefore,

$$P\begin{pmatrix}\text{less than 3 or} \\ \text{greater than 7}\end{pmatrix} = \frac{2}{10} + \frac{3}{10} - 0 = \frac{5}{10} = \frac{1}{2}$$

Mutually Exclusive

- Two events *A* and *B* are **mutually exclusive** if it is impossible for both events to occur simultaneously.

Example

- One card is selected from a standard deck of playing cards. Determine the probability of the following events.

  a) selecting a 3 or a jack

  b) selecting a jack or a heart

  c) selecting a picture card or a red card

  d) selecting a red card or a black card

Solutions

a) 3 or a jack
(mutually exclusive)

$$P(3) + P(\text{jack}) = \frac{4}{52} + \frac{4}{52} = \frac{8}{52} = \frac{2}{13}$$

b) jack or a heart

$$P(\text{jack}) + P(\text{heart}) - P\begin{pmatrix}\text{jack and} \\ \text{heart}\end{pmatrix} = \frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{13}$$

Solutions continued

c) picture card or red card

$$P(\text{picture}) + P(\text{red}) - P\begin{pmatrix}\text{picture \&} \\ \text{red card}\end{pmatrix} = \frac{12}{52} + \frac{26}{52} - \frac{6}{52}$$

$$= \frac{32}{52} = \frac{8}{13}$$

d) red card or black card

(mutually exclusive)

$$P(\text{red}) + P(\text{black}) = \frac{26}{52} + \frac{26}{52}$$

$$= \frac{52}{52} = 1$$

And Problems

- $P(A \text{ and } B) = P(A) \bullet P(B)$
- Example: Two cards are to be selected *with replacement* from a deck of cards. Find the probability that two red cards will be selected.

$$P(A) \cdot P(B) = P(\text{red}) \cdot P(\text{red})$$

$$= \frac{26}{52} \cdot \frac{26}{52}$$

$$= \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

Example

- Two cards are to be selected *without replacement* from a deck of cards. Find the probability that two red cards will be selected.

$$P(A) \cdot P(B) = P(\text{red}) \cdot P(\text{red})$$
$$= \frac{26}{52} \cdot \frac{25}{51}$$
$$= \frac{1}{2} \cdot \frac{25}{51} = \frac{25}{102}$$

Independent Events

- Event A and Event B are **independent events** if the occurrence of either event in no way affects the probability of the occurrence of the other event.
- Experiments done with replacement will result in independent events, and those done without replacement will result in dependent events.

Example

- A package of 30 tulip bulbs contains 14 bulbs for red flowers, 10 for yellow flowers, and 6 for pink flowers. Three bulbs are randomly selected and planted. Find the probability of each of the following.
  a. All three bulbs will produce pink flowers.
  b. The first bulb selected will produce a red flower, the second will produce a yellow flower and the third will produce a red flower.
  c. None of the bulbs will produce a yellow flower.
  d. At least one will produce yellow flowers.

Solution

- 30 tulip bulbs, 14 bulbs for red flowers, 10 for yellow flowers, and 6 for pink flowers.

a. All three bulbs will produce pink flowers.

$$P(\text{3 pink}) = P(\text{pink 1}) \cdot P(\text{pink 2}) \cdot P(\text{pink 3})$$

$$= \frac{6}{30} \cdot \frac{5}{29} \cdot \frac{4}{28}$$

$$= \frac{1}{203}$$

Solution (continued)

- 30 tulip bulbs, 14 bulbs for red flowers, 10 for yellow flowers, and 6 for pink flowers.

b. The first bulb selected will produce a red flower, the second will produce a yellow flower and the third will produce a red flower.

$$P(\text{red, yellow, red}) = P(\text{red}) \cdot P(\text{yellow}) \cdot P(\text{red})$$
$$= \frac{14}{30} \cdot \frac{10}{29} \cdot \frac{13}{28}$$
$$= \frac{13}{174}$$

Solution (continued)

- 30 tulip bulbs, 14 bulbs for red flowers, 10 for yellow flowers, and 6 for pink flowers.

c. None of the bulbs will produce a yellow flower.

$$P\begin{pmatrix}\text{none}\\ \text{yellow}\end{pmatrix} = P\begin{pmatrix}\text{first not}\\ \text{yellow}\end{pmatrix} \cdot P\begin{pmatrix}\text{second not}\\ \text{yellow}\end{pmatrix} \cdot P\begin{pmatrix}\text{third not}\\ \text{yellow}\end{pmatrix}$$
$$= \frac{20}{30} \cdot \frac{19}{29} \cdot \frac{18}{28}$$
$$= \frac{57}{203}$$

Solution (continued)

- 30 tulip bulbs, 14 bulbs for red flowers, 10 for yellow flowers, and 6 for pink flowers.

d. At least one will produce yellow flowers.

$$P(\text{at least one yellow}) = 1 - P(\text{no yellow})$$
$$= 1 - \frac{57}{203}$$
$$= \frac{146}{203}$$

Examples

- A couple has three children. Assuming independence and that the probability of a boy is ½, determine the probability that
  A. All three children are girls.
  B. All three children are boys.
  C. The youngest child is a boy and the two older children are girls.
  D. The youngest child is a girl, the middle child is a boy, and the oldest child is a girl.

Examples

- Each question of a five-question multiple-choice exam has four possible answers. Sam picks an answer at random for each question. Determine the probability that he selects the correct answer on
  A. Any one question.
  B. Only the first question.
  C. Only the third and fourth questions
  D. All five questions.
  E. None of the questions
  F. At least one of the questions.