Section 8.4

Dimensional Analysis and Conversions to and from the Metric System

Dimensional Analysis

- *Dimensional analysis* is a procedure used to convert from one unit of measurement to a different unit of measurement.
- A *unit fraction* is any fraction in which the numerator and denominator contain different units and the value of the fraction is 1.
- Examples of unit fractions:

$$\frac{16\text{ oz}}{1\text{ lb}} \qquad \frac{1\text{ hr}}{60\text{ min}} \qquad \frac{12\text{ in.}}{1\text{ ft}}$$

U.S. Customary Units

U.S. Customary Units	
1 foot = 12 inches	1 quart = 2 pints
1 yard = 3 feet	1 gallon = 4 quarts
1 mile = 5280 feet	1 minute = 60 seconds
1 pound = 16 ounces	1 hour = 60 minutes
1 ton = 2000 pounds	1 day = 24 hours
1 cup (liquid) = 8 fluid ounces	
1 pint = 2 cups	1 year = 365 days

Example: Using Dimensional Analysis

- A recipe calls for 8 cups of blueberries. How many pints is this?

Solution:

$$8 \text{ cups} = (8 \cancel{\text{cups}})\left(\frac{1 \text{ pint}}{2 \cancel{\text{cups}}}\right) = 4 \text{ pints}$$

- Convert 75 miles per hour to inches per minute.

Solution:

$$75\frac{\text{mi}}{\text{hr}} = \left(75\frac{\cancel{\text{mi}}}{\cancel{\text{hr}}}\right)\left(\frac{5280\cancel{\text{ft}}}{1\cancel{\text{mi}}}\right)\left(\frac{12 \text{ in}}{1 \cancel{\text{ft}}}\right)\left(\frac{1 \cancel{\text{hr}}}{60 \text{ min}}\right)$$

$$= \frac{(75)(5280)(12)}{60}\frac{\text{in}}{\text{min}} = 79{,}200\frac{\text{in}}{\text{min}}$$

Conversion to and from the Metric System - Length

LENGTH
U.S. to Metric
1 inch (in.) ≈ 2.54 centimeters (cm)
1 foot (ft) ≈ 30.48 centimeters (cm)
1 yard (yd) ≈ 0.91 meter (m)
1 mile (mi) ≈ 1.61 kilometers (km)

Conversion to and from the Metric System - Area

AREA
U.S. to Metric
1 square inch (in.^{2}) ≈ 6.5 square centimeters (cm^2)
1 square foot (ft^2) ≈ 0.09 square meter (m^2)
1 square yard (yd^2) ≈ 0.8 square meter (m^2)
1 square mile (mi^2) ≈ 2.6 square kilometers (km^2)
1 acre ≈ 0.4 hectare (ha)

Conversion to and from the Metric System - Volume

VOLUME
U.S. to Metric
1 teaspoon (tsp) ≈ 5 milliliters (mℓ)
1 tablespoon (tbsp) ≈ 15 milliliters (mℓ)
1 fluid ounce (fl oz) ≈ 30 milliliters (mℓ)
1 cup (c) ≈ 0.24 liter (ℓ)
1 pint (pt) ≈ 0.47 liter (ℓ)

Conversion to and from the Metric System - Volume

VOLUME
U.S. to Metric
1 quart (qt) ≈ 0.95 liter (ℓ)
1 gallon (gal) ≈ 3.8 liters (ℓ)
1 cubic foot (ft^3) ≈ 0.03 cubic meter (m^3)
1 cubic yard (yd^3) ≈ 0.76 cubic meter (m^3)

Conversion to and from the Metric System - Weight (Mass)

WEIGHT OR MASS
U.S. to Metric
1 ounce (oz) ≈ 28 grams (g)
2.2 pounds (lb) ≈ 1 kilogram (kg)
1 ton (T) ≈ 0.9 tonne (t)

Example: Volume and Area

- A gas tank holds 22.6 gallons of gas. How many liters is this?

Solution:

$$22.6 \text{ gal} \times \frac{3.8\ \ell}{\text{gal}} = 85.88\ \ell$$

- The area of a box is 14.25 in^2. What is its area in square centimeters?

Solution:

$$14.25 \text{ in}^2 \left(\frac{6.5 \text{ cm}^2}{1 \text{ in}^2} \right) = 92.625 \text{ cm}^2$$

Example: Converting Speed

A road in Toronto, Canada shows that the speed limit is 62 kph. Determine the speed in miles per hour.

Solution:

$$62\text{ km}\left(\frac{1\text{ mi}}{1.6\text{ km}}\right)=\frac{62}{1.6}\text{mi}=38.75\text{ mi}$$

- Since 62 km equals 38.75 mi, 62 kph is equivalent to 38.75 mph.

Example: Weight (Mass) Conversion for Medication

A newborn baby weighs 8 pounds 4 ounces. If 20 mg of a medication is given for each kilogram of the baby's weight, what dosage should be given?

Solution:

$$8\text{ lbs}\left(\frac{16\text{ oz}}{1\text{ lb}}\right)+4\text{ oz}=128\text{ oz}+4\text{ oz}=132\text{ oz}$$

$$132\text{ oz}\left(\frac{28\text{ g}}{\text{oz}}\right)\left(\frac{1\text{ kg}}{1000\text{ g}}\right)\left(\frac{20\text{ mg}}{1\text{ kg}}\right)=73.92\text{ mg}$$

The dosage of the medication is 73.92 mg.