

PRACTICE Exam 1

Math 120, Math for General Education

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100 points

Show all work to receive full credit. You may use a calculator. CHECK YOUR WORK!!!!

NAME Answer Key

1. (4 pts) Determine how many distinct cases must be listed in the truth table for the compound statement.

$$\sim(p \vee q) \vee (q \wedge \sim r)$$

3 STATEMENTS \Rightarrow 8 CASES
 $2^n = 2^3 = 8$

2. (10 pts) Translate the statement into symbols then construct a truth table.

Let p = The cab is late.

q = The plane takes off on time.

r = Nancy has her plane ticket.

$$(p \vee \sim q) \wedge \sim r$$

The cab is late or the plane does not take off on time, and Nancy does not have her ticket.

p	q	r	(p	v	~q)	^	~r		
T	T	T	T	T	F	F	F		
T	T	F	T	T	F	T	T		
T	F	T	T	T	T	F	F		
T	F	F	T	T	T	T	T		
F	T	T	F	F	F	F	F		
F	T	F	F	F	F	F	T		
F	F	T	F	T	T	F	F		
F	F	F	F	T	T	T	T		
			(1)	(4)	(2)	(5)	(3)		

Use the table above to construct your truth table. Put in the appropriate symbols in the top row for the expression that you are evaluating. There may be more columns than needed.

3. (10 pts) Construct a truth table for the following statement: $\sim(p \wedge s) \rightarrow (p \rightarrow (\sim q \wedge s))$

p	q	s	~	(p	^	s)	→	(p	→	(~q	^	s)
T	T	T	F	T	T	T	T	T	F	F	F	T
T	T	F	T	T	F	F	F	T	F	F	F	F
T	F	T	F	T	T	T	T	T	T	T	T	T
T	F	F	T	T	F	F	F	T	F	T	F	F
F	T	T	T	F	F	T	T	F	T	F	F	T
F	T	F	T	F	F	F	T	F	T	F	F	F
F	F	T	T	F	F	T	T	F	T	T	T	T
F	F	F	T	F	F	F	T	F	T	T	F	F
			(9)	(1)	(7)	(2)	(10)	(3)	(8)	(5)	(6)	(4)

For Problems 4 & 5: Given the following definitions, write the compound statements in symbols.

Let r = The food is good.

p = I eat too much.

q = I'll exercise.

4. (4 pts) If the food is good and if I eat too much, then I'll exercise.

$$(r \wedge p) \rightarrow q$$

5. (4 pts) If I exercise, then the food won't be good and I won't eat too much.

$$q \rightarrow (\sim r \wedge \sim p)$$

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6. (5 pts) Given p is true, q is true, and r is false, find the truth value of the statement.

$$\begin{aligned} & (\sim p \leftrightarrow \sim q) \wedge (p \leftrightarrow \sim r) \\ & (F \leftrightarrow F) \wedge (T \leftrightarrow T) \\ & T \wedge T \\ & T \end{aligned}$$

7. (8 pts) Use DeMorgan's laws or a truth table to determine whether the two statements are equivalent.

$$\sim(p \wedge q), \sim p \wedge \sim q$$

$$\sim(p \wedge q) \Leftrightarrow \sim p \vee \sim q \quad \text{ACCORDING TO DeMORGAN'S LAWS}$$

$$\sim p \vee \sim q \neq \sim p \wedge \sim q$$

NOT EQUIVALENT

8. (5 pts) Write the contrapositive of the following statement:

If the chores are done, then we will go to the carnival and we will eat cotton candy.

IF WE DO NOT GO TO THE CARNIVAL & WE DO NOT EAT COTTON CANDY, THEN THE CHORES ARE NOT DONE.

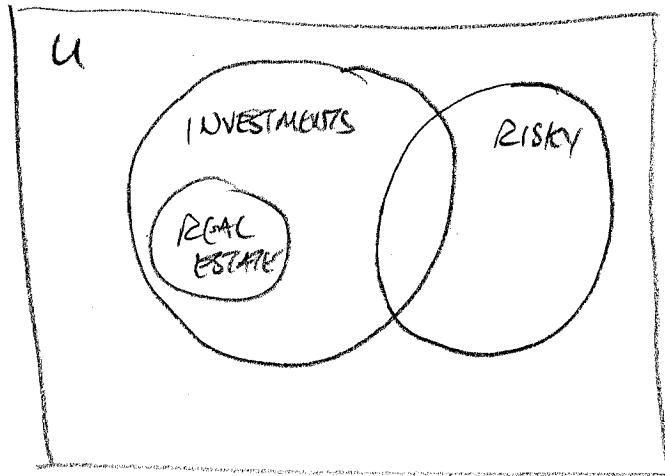
For Problems 9 and 10 (5 pts each), construct an Euler Diagram to determine whether the syllogism is valid.

9. Some investments are risky.

Real estate is an investment.

=> Real estate is risky.

NOT VALID

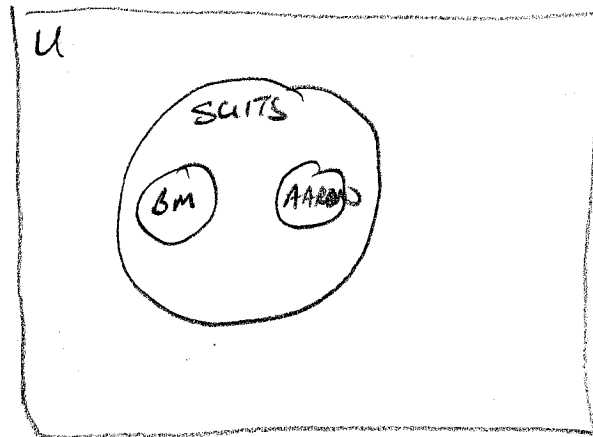


10. All businessmen wear suits.

Aaron wears a suit.

=> Aaron is a businessman.

NOT VALID



Determine whether or not the first number is divisible by the second number (4 pts each)

11. 797,886; 3 YES, SUM OF DIGITS = 45 DIVISIBLE BY 3

12. 561,897; 9 YES, SUM OF DIGITS = 36 DIVISIBLE BY 9

13. 19,290; 6 YES, EVEN => DIVISIBLE BY 2
SUM = 21 - DIVISIBLE BY 3

14. (5 pts) Find the prime factorization of 684.

	684
2	342
2	171
3	57
3	19
19	

$$= 2^2 \cdot 3^2 \cdot 19$$

15. (5 pts) Find the least common multiple (LCM) of 45, 28, and 150.

	45	28	150
2	45	14	75
2	45	7	75
3	15	7	25
3	5	7	25
5	5	7	5
5	1	7	1
7	1	1	1

$$LCM = 2^2 \cdot 3^2 \cdot 5^2 \cdot 7$$

$$= 6300$$

16. (5 pts) Find a_5 for the geometric sequence

$$a_1 = 3, r = 6$$

$$n = 5$$

$$a_n = a_1 r^{(n-1)} \quad a_n = 3(6)^{(5-1)} = 3 \cdot 6^4$$

$$= 3 \cdot 1296$$

$$= 3888$$

(Additional: write the first 5 terms of the sequence)

$$a_1 = 3$$

$$a_2 = 3(6)^1 = 18$$

$$a_3 = 3(6)^2 = 3(36) = 108$$

$$a_4 = 3(6)^3 = 3(216) = 648$$

$$a_5 = 3(6)^4 = 3(1296) = 3888$$

$$\{ 3, 18, 108, 648, 3888 \}$$

17. (5 pts) Find a_{26} for the arithmetic sequence

$a_1 = -8, d = -1/5$

$a_n = a_1 + (n-1)d$

$a_{26} = -8 + (26-1)(-1/5)$
 $= -8 + (25)(-1/5) = \boxed{-13}$

(Additional: write the first 5 terms of the sequence)

$a_2 = -8 - 1/5 = -8 1/5$

$a_5 = -8 + 4(-1/5)$

$a_3 = -8 + 2(-1/5) = -8 2/5$

$= -8 4/5$

$a_4 = -8 + 3(-1/5) = -8 3/5$

$\{-8, -8 1/5, -8 2/5, -8 3/5, -8 4/5\}$

For Problems 18 & 19, determine whether the sequence is a Fibonacci-type sequence. If it is, determine the next 3 terms of the sequence. (4 pts each)

18. -9, 4, -5, -1, -6, ...

YES -7, -13, -20

19. -7, 3, -4, -11, -15, ...

NO

$a_4 \neq a_3 + a_2$

BONUS (total of 5 extra points)

Determine which, if any, of the three statements are equivalent.

- a) The car has leather or I will buy it.
- b) If the car has leather, then I will buy it.
- c) The car does not have leather or I will buy it

$P = \text{CAR HAS LEATHER}$
 $Q = \text{I WILL BUY IT}$

- A) $(P \vee Q)$
- B) $(P \Rightarrow Q)$
- C) $(\sim P \vee Q)$

P	Q	$(P \vee Q)$	$(P \Rightarrow Q)$	$(\sim P \vee Q)$
T	T	T	T	T
T	F	T	F	T
F	T	T	T	T
F	F	F	T	T

B & C ARE EQUIVALENT