

3.1 Statements and Logical Connectives

Math 120

Math for General Education

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History of Logic

◆ Ancient Greeks

- **1st to analyze ways humans arrive at conclusions**
- **Aristotle organized the study of logic first.**

◆ Gottfried Leibnitz (17th Century)

- **Symbolic logic**
- **Used symbols to represent written statements**

History of Logic

- ◆ **George Boole (19th Century)**
 - **Father of Symbolic Logic**
- ◆ **Logic studied to exercise mind's ability to reason**
 - **Understanding logic enables one to**
 - ◆ **Think clearly**
 - ◆ **Communicate effectively**
 - ◆ **Make more convincing arguments**
 - ◆ **Develop reasoning patterns**
- ◆ **Abbot and Costello "Who's on First?"**

Logic & English Language

- ◆ **Reading, writing, & speaking use "and", "or", "if ...then..." to connect thoughts.**
- ◆ **In logic, these are called connectives.**

Exclusive VS. Inclusive "or"

- ◆ **Exclusive "or"**
 - **One or the other, but not both**
 - "I sentence you to 6 months in prison or 10 months of community service."
- ◆ **Inclusive "or"**
 - **One or the other, or both**
 - "Would you like a cup of soup or a sandwich?"
 - 3 possibilities
 - ◆ Cup of soup
 - ◆ Sandwich
 - ◆ Both

Statements & Logical Connections

- ◆ **Statement – sentence that can be judged as either true or false.**
 - **Labeling "true" or "false" – assigning a "truth value".**
- 1) Sea World is in San Francisco
 - 2) The Grand Canyon is in Arizona
 - 3) San Diego's football team is called the Seahawks.

Quantifiers

- ◆ Sometimes it is necessary to change statements to its opposite meaning – use “negation”.
 - Negation of true statement → Always False
 - Negation of false statement → Always True
 - Be careful with negating statements with “all”, “none” (or “no”), and “some”

Quantifier Example

- ◆ No birds can swim. – True or False?
- ◆ Negation? – Therefore the neg must be true.
 - ◆ “All birds can swim.”
Careful: also false so will not work!!!
 - ◆ At least one bird can swim
 - or
 - ◆ Some birds can swim

Negation of Quantified Statements

<u>Form of Statement</u>	<u>Form of Negation</u>
All are.	Some are not.
None are.	Some are.
Some are.	None are.
Some are not.	All are.

Writing Negations:

- 1) **Some calculators are graphing calculators.**
No calculators are graphing calculators
-

- 2) **All cars are red.**
Some cars are not red.
-

Or
At least 1 car is not red

Compound Statements:

- ◆ Statements consisting of 2 or more simple statements.
- ◆ Often use **connectives** to join 2 simple statements
 - **"and"**
 - **"or"**
 - **"if ... then ..."**
 - **"if and only if"**
- ◆ Use **letters** to designate statements instead of rewriting over and over again.
- ◆ Typically (but not limited to) use **p, q, r, and s** to represent simple statements

Not Statements

- ◆ **Negation symbol \sim - read as "not"**
 - Statement: Jason is a college student.
 - Negation: **Jason is not a college student.**

Using ALL Symbols:

Let: p = Jason is a college student.

Then: $\sim p$ = **Jason is not a college student.**
(compound statement)

$\sim(\sim p)$ = **p** = **Jason is a college student**

Not Statements

Statement - Edgar is not at home.

- **Already a negation: $\sim p$**

Then $p =$ **Edgar is at home**.

Convention:

- p, q, r, s represent statements that are not negated.

- negated statements use the " \sim " symbol

$\sim p, \sim q, \sim r, \sim s$

AND

◆ **Conjunction** - \wedge read "and"

Ex: Martha Stewart -

Let p : You will serve 5 months in prison.

q : You will pay a \$30,000 fine

Writing in symbolic form:

You will serve 10
months in prison.

↑
 p

and

↑
 \wedge

You will pay a
\$30,000 fine.

↑
 q

AND

- The box is big, but it is not heavy

- p: Box is big

- q: Box is heavy

$p \wedge \sim q$

OR

◆ Disjunction – \vee read “or”

◆ Remember: “or” is inclusive unless otherwise stated

Let p: Leanne will go to the beach.

q: Leanne will go to the Padres game

1) Leanne will go to the beach or Leanne will go to the Padres game.

$p \vee q$

2) Leanne will go to the Padres game or Leanne will not go to the beach.

$q \vee \sim p$

3) Leanne will not go to the beach or Leanne will not go to the Padres game.

$\sim p \vee \sim q$

OR

◆ OR Inclusive

- ◆ "Leanne will go to the beach or Leanne will go to the Padres game" may mean

1. Leanne will go to the beach, or
2. Leanne will go to the Padres game, or
3. Leanne will go to both the beach and the Padres game.

Compound Statements (more)

◆ Compound statements containing more than one connective.

- Commas are used to indicate which statements are grouped together
- Symbolically, simple statements on same side of comma are: grouped together within ().

Compound Statements (more)

"Salvador Dali was an artist (p) or Robert Redford is an actor (q), and Boise is in Idaho (r)."

$$(p \vee q) \wedge r$$

"Salvador Dali was an artist (p), or Robert Redford is an actor (q) and Boise is in Idaho (r)."

$$p \vee (q \wedge r)$$

Compound Statements (more)

p : Dinner includes soup

q : Dinner includes salad

r : Dinner includes the vegetable of the day

A. Dinner includes soup, and salad or the vegetable of the day.

$$p \wedge (q \vee r)$$

B. Dinner includes soup and salad, or the vegetable of the day.

$$(p \wedge q) \vee r$$

Type of Compound Statement

Statement	Symbolic Representation	Type of Statement
Dinner includes soup, and salad or the vegetable of the day.	$p \wedge (q \vee r)$	Conjunction
Dinner includes soup and salad, or the vegetable of the day	$(p \wedge q) \vee r$	Disjunction

Negation (more)

Negation symbol (\sim) has the effect of: _____
negating *only* the statement that directly follows it.

Let p : Joseph is arresting a perp.

q : Pat is delivering pizzas.

$p \wedge \sim q$ Joseph is arresting a perp and Pat is not delivering pizzas.

$\sim p \vee \sim q$ Joseph is not arresting a perp or Pat is not delivering pizzas

$\sim(p \wedge q)$ It is not true that Joseph is arresting a perp and Pat is delivering pizzas.
 (Can also be read as "It is false that....")

Conditional

◆ Symbolized by \longrightarrow & read "if – then"

◆ $p \rightarrow q$ reads: "if p, then q"

Antecedent: precedes the arrow

Consequent: follows the arrow

$$\sim(p \vee q) \rightarrow (p \wedge q)$$

◆ **Conditional symbol** may be placed between any 2 statements even if statements are not related.

- Word "**then**" sometimes not explicitly stated

"If you pass this course, I will buy you a car"

- conditional that actually means

"If you pass this course, then I will buy you a car."

Conditional

Let p: Carolyn goes to the library.

q: Carolyn will study.

A. If Carolyn goes to the library, then she will study. $p \rightarrow q$

B. If Carolyn does not go to the library, then she will not study. $\sim p \rightarrow \sim q$

C. It is false that if Carolyn goes to the library, then she will study. $\sim(p \rightarrow q)$

If and Only If Statements

Biconditional statements symbolized by \leftrightarrow is read "**If and only if**" (sometimes abbreviated as IFF).

$p \leftrightarrow q$ reads: "**p if and only if q**"

Let p : The Web browser is working.

q : The computer is connected to the Internet.

$q \leftrightarrow p$

The computer is connected to the Internet if & only if the Web browser is working.

$\sim(p \leftrightarrow \sim q)$

It is false that the Web browser is working if and only if the computer is not connected to the Internet.

Dominance of Connectives

Least dominant 1. Negation, \sim Evaluate first

2. Conjunction, \wedge
Disjunction, \vee

3. Conditional, \rightarrow

Most dominant 4. Biconditional, \leftrightarrow Evaluate last

$p \wedge q \vee r$ – conjunction or disjunction - use grouping symbols ()

- if no (), evaluate least dominant first and most dominant last.

Example Connective Types

Statement	Most Dominant connective used	Statement means	Type of statement
$\sim p \vee q$	\vee	$(\sim p) \vee q$	Disjunction
$p \rightarrow q \vee r$	\rightarrow	$p \rightarrow (q \vee r)$	Conditional
$p \wedge q \rightarrow r$	\rightarrow	$(p \wedge q) \rightarrow r$	Conditional
$p \rightarrow q \leftrightarrow r$	\leftrightarrow	$(p \rightarrow q) \leftrightarrow r$	Biconditional
$p \vee r \leftrightarrow r \rightarrow \sim p$	\leftrightarrow	$(p \vee r) \leftrightarrow (r \rightarrow \sim p)$	Biconditional
$P \rightarrow r \leftrightarrow s \wedge p$	\leftrightarrow	$(p \rightarrow r) \leftrightarrow (s \wedge p)$	Biconditional

Practice Problems

◆ Page 110-111 # 12, 32, 38, 50

◆ HOMEWORK: Pages 109 – 112

#1 – 8 all

#9 – 75 multiples of 3

(9, 12, 15, 18, 21,75)