3.4 Equivalent Statements

Math 120

Math for General Education

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Equivalent Statements

- Two statements are **equivalent**, symbolized ⇔, if both statements have **exactly the same truth values** in the answer columns of the truth tables.
- Determine if the two compound statements are equivalent:

p ∧ (q ∨ r) and **(p ∧ q) ∨ (p ∧ r)**

*Therefore: p Λ (q V r)⇔(p Λ q) V (p Λ r)

Equivalent Statements (cont.)

#			1	3	2	4	6	5
p	q	r	p	Λ	(q V r)	(p Λ q)	V	(p Λ r)
T	T	T	T	T	T	T	T	T
T	T	F	T	T	T	T	T	F
T	F	T	T	T	T	F	T	T
T	F	F	T	F	F	F	F	F
F	T	T	F	F	T	F	F	F
F	T	F	F	F	T	F	F	F
F	F	T	F	F	T	F	F	F
F	F	F	F	F	F	F	F	F

Equivalent Statements(cont.)

- **Determine which statement is logically equivalent to "It is not true that the tire is both out of balance and flat." ~(p Λ q)**

a) If the tire is not flat, then the tire is not out of balance.
~q → ~p

b) The tire is not out of balance or the tire is not flat.
~p Λ ~ q

c) The tire is not flat and the tire is not out of balance
~q Λ ~p

d) If the tire is not out of balance, then the tire is not flat.
~p → ~q

p: **The tire is out of balance.**

q: **The tire is flat.**

Equivalent Statements(cont.)

						a	b	c	d
p	q	~p	~q	~	(p ∧ q)	~q→ ~p	~p ∨~q	~q∧ ~p	~p→ ~q
T	T	F	F	F	T	T	F	F	T
T	F	F	T	T	F	F	T	F	T
F	T	T	F	T	F	T	T	F	F
F	F	T	T	T	F	T	T	T	T

Therefore: Choice (b) is equivalent "The tire is not out of balance or the tire is not flat."

De Morgan's Laws

- **Two special laws named after Augustus De Morgan, an English mathematician**
  1. **~(p ∧ q) ⇔ ~p ∨ ~q**
  2. **~(p ∨ q) ⇔ ~p ∧ ~q**

*You can prove these true on your own by constructing the truth tables and checking.

Using De Morgan's Laws

- Write a statement that is logically equivalent to:
  - "It is not true that tomatoes are poisonous or eating peppers cures the common cold."

p: Tomatoes are poisonous.

q: Eating peppers cures the common cold.

Symbols: ~(p V q)

Apply De Morgans: ~p Λ ~ q

New Statement: "Tomatoes are not poisonous and eating peppers does not cure the common cold."

Negation of the Conditional Statement

- From page 141: p →q ⇔ ~p V q
- We use this plus De Morgan's Laws to negate a conditional statement.

Example: Find the equivalent to: ~(p → q)

Begin with : p → q ⇔ ~p V q

Negate both statements: ~(p → q) ⇔ ~(~p V q)

Use De Morgan's laws (left): ~(p → q) ⇔ p Λ ~q

Thus, ~(p→q) is equivalent to p Λ ~q.

Variations of the Conditional Statement

- The variations of the conditional statement are made by switching and/or negating the antecedent and the consequent of a conditional statement. The are all equivalent to each other.

NAME	SYMBOLIC FORM	READ
Conditional	$p \rightarrow q$	"If p, then q"
Converse of conditional	$q \rightarrow p$	"If q, then p"
Inverse of conditional	$\sim p \rightarrow \sim q$	"If not p, then not q"
Contrapositive of conditional	$\sim q \rightarrow \sim p$	"If not q, then not p"

Variations (cont.)

Using the statement: *"If the dryer is making a loud noise, then you need to replace the blower fan."* $p \rightarrow q$

Write the converse, the inverse, and the contrapositive.

$q \rightarrow p$
Converse: "If the you need to replace the blower fan, then the dryer is making a loud noise."

$\sim p \rightarrow \sim q$
Inverse: "If the dryer is not making a loud noise, then you do not need to replace the blower fan."

$\sim q \rightarrow \sim p$
Contrapositive: "If you do not need to replace the blower fan, then the dryer is not making a loud noise."

Practice Problems

- **In Class Practice**
  - Pages 147-148
  - #16, 35, 65
- **Homework**
  - Pages 147 – 149
  - #9 – 69 multi of 3