Chapter 4
Probability

4-1 Review and Preview

4-2 Basic Concepts of Probability

4-3 Addition Rule

4-4 Multiplication Rule: Basics

4-5 Multiplication Rule: Complements and Conditional Probability

4-6 Counting

Basic Rules for Computing Probability

Rule 1: Relative Frequency Approximation of Probability

Conduct (or observe) a procedure, and count the number of times event *A* actually occurs. Based on these actual results, *P*(*A*) is approximated as follows:

$$P(A) = \frac{\text{\# of times } A \text{ occurred}}{\text{\# of times procedure was repeated}}$$

Basic Rules for Computing Probability - continued

Rule 2: Classical Approach to Probability (Requires Equally Likely Outcomes)

Assume that a given procedure has *n* different simple events and that each of those simple events has an equal chance of occurring. If event *A* can occur in *s* of these *n* ways, then

$$P(A) = \frac{s}{n} = \frac{\text{number of ways } A \text{ can occur}}{\text{number of different simple events}}$$

Compound Event

Formal Addition Rule

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

where $P(A \text{ and } B)$ denotes the probability that A and B both occur at the same time as an outcome in a trial of a procedure.

Complementary Events

$P(A)$ and $P(\bar{A})$ are disjoint

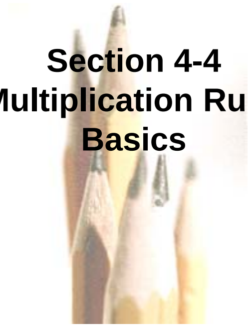
It is impossible for an event and its complement to occur at the same time.

Rule of Complementary Events

$$P(A) + P(\bar{A}) = 1$$

$$P(\bar{A}) = 1 - P(A)$$

$$P(A) = 1 - P(\bar{A})$$



Section 4-4 Multiplication Rule: Basics

Key Concept

The basic multiplication rule is used for finding *P*(*A* and *B*), the probability that event *A* occurs in a first trial and event *B* occurs in a second trial.

If the outcome of the first event *A* somehow affects the probability of the second event *B*, it is important to adjust the probability of *B* to reflect the occurrence of event *A*.

Notation

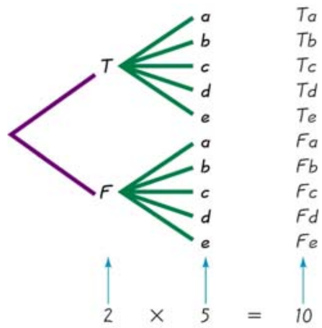
$P(A \text{ and } B)$ =

P(event A occurs in a first trial and event B occurs in a second trial)

Tree Diagrams

This figure summarizes the possible outcomes for a true/false question followed by a multiple choice question.

Note that there are 10 possible combinations.



Conditional Probability Key Point

We must adjust the probability of the second event to reflect the outcome of the first event.

Conditional Probability Important Principle

The probability for the second event *B* should take into account the fact that the first event *A* has already occurred.

Notation for Conditional Probability

***P*(*B*|*A*) represents the probability of event *B* occurring after it is assumed that event *A* has already occurred (read *B*|*A* as "*B* given *A*.")**

Dependent and Independent

Two events *A* and *B* are independent if the occurrence of one does not affect the *probability* of the occurrence of the other. (Several events are similarly independent if the occurrence of any does not affect the probabilities of the occurrence of the others.) If *A* and *B* are not independent, they are said to be dependent.

Dependent Events

Two events are dependent if the occurrence of one of them affects the *probability* of the occurrence of the other, but this does not necessarily mean that one of the events is a *cause* of the other.

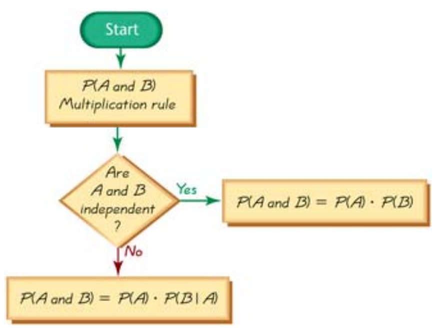
Formal Multiplication Rule

- $P(A \text{ and } B) = P(A) \cdot P(B|A)$
- Note that if A and B are independent events, P(B|A) is really the same as P(B).

Intuitive Multiplication Rule

When finding the probability that event *A* occurs in one trial and event *B* occurs in the next trial, multiply the probability of event *A* by the probability of event *B*, but be sure that the probability of event *B* takes into account the previous occurrence of event *A*.

Applying the Multiplication Rule



Applying the Multiplication Rule

Caution

When applying the multiplication rule, always consider whether the events are independent or dependent, and adjust the calculations accordingly.

Multiplication Rule for Several Events

In general, the probability of any sequence of independent events is simply the product of their corresponding probabilities.

Summary of Fundamentals

- In the addition rule, the word "or" in *P*(*A* or *B*) suggests addition. Add *P*(*A*) and *P*(*B*), being careful to add in such a way that every outcome is counted only once.
- In the multiplication rule, the word "and" in *P*(*A* and *B*) suggests multiplication. Multiply *P*(*A*) and *P*(*B*), but be sure that the probability of event *B* takes into account the previous occurrence of event *A*.

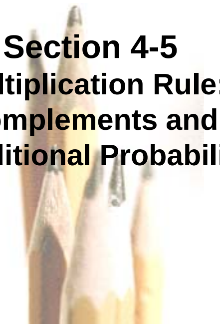
Recap

In this section we have discussed:

- Notation for *P*(*A* and *B*).
- Tree diagrams.
- Notation for conditional probability.
- Independent events.
- Formal and intuitive multiplication rules.

Examples

Pg 168: 7, 12, 18, 21, 23



Section 4-5
Multiplication Rule: Complements and Conditional Probability

Complements: The Probability of "At Least One"

- "At least one" is equivalent to "one or more."
- The complement of getting at least one item of a particular type is that you get no items of that type.

Finding the Probability of "At Least One"

To find the probability of at least one of something, calculate the probability of none, then subtract that result from 1. That is,

***P*(at least one) = 1 – *P*(none).**

Conditional Probability

A conditional probability of an event is a probability obtained with the additional information that some other event has already occurred. $P(B|A)$ denotes the conditional probability of event B occurring, given that event A has already occurred, and it can be found by dividing the probability of events A and B both occurring by the probability of event A:

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$

Intuitive Approach to Conditional Probability

The conditional probability of B given A can be found by assuming that event A has occurred, and then calculating the probability that event B will occur.

Confusion of the Inverse

To incorrectly believe that $P(A|B)$ and $P(B|A)$ are the same, or to incorrectly use one value for the other, is often called confusion of the inverse.

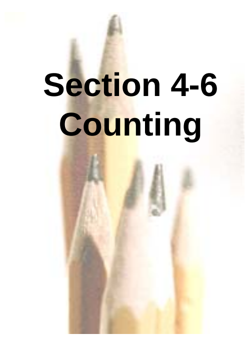
Recap

In this section we have discussed:

- ❖ Concept of "at least one."
- ❖ Conditional probability.
- ❖ Intuitive approach to conditional probability.

Examples

Pages 176-178: 10, 15, 20, 22, 25, 30



Section 4-6 Counting

Key Concept

In many probability problems, the big obstacle is finding the total number of outcomes, and this section presents several methods for finding such numbers without directly listing and counting the possibilities.

Fundamental Counting Rule

For a sequence of two events in which the first event can occur *m* ways and the second event can occur *n* ways, the events together can occur a total of *m*•*n* ways.

Notation

The factorial symbol ! denotes the product of decreasing positive whole numbers.

For example,

$$4! = 4 \bullet 3 \bullet 2 \bullet 1 = 24.$$

By special definition, 0! = 1.

Factorial Rule

A collection of n different items can be arranged in order $n!$ different ways. (This factorial rule reflects the fact that the first item may be selected in n different ways, the second item may be selected in $n - 1$ ways, and so on.)

Permutations Rule
(when items are all different)

Requirements:

1. There are n different items available. (This rule does not apply if some of the items are identical to others.)
2. We select r of the n items (without replacement).
3. We consider rearrangements of the same items to be different sequences. (The permutation of ABC is different from CBA and is counted separately.)

If the preceding requirements are satisfied, the number of permutations (or sequences) of r items selected from n available items (without replacement) is

$$_nP_r = \frac{n!}{(n-r)!}$$

Permutations Rule
(when some items are identical to others)

Requirements:

1. There are n items available, and some items are identical to others.
2. We select all of the n items (without replacement).
3. We consider rearrangements of distinct items to be different sequences.

If the preceding requirements are satisfied, and if there are n_1 alike, n_2 alike, . . . n_k alike, the number of permutations (or sequences) of all items selected without replacement is

$$\frac{n!}{n_1! \, . \, n_2! \, .. \, . \, . \, . \, . \, . \, . \, . \, n_k!}$$

Combinations Rule

Requirements:

1. There are n different items available.
2. We select r of the n items (without replacement).
3. We consider rearrangements of the same items to be the same. (The combination of ABC is the same as CBA.)

If the preceding requirements are satisfied, the number of combinations of r items selected from n different items is

$$_nC_r = \frac{n!}{(n - r)!\, r!}$$

Permutations versus Combinations

When different orderings of the same items are to be counted separately, we have a permutation problem, but when different orderings are not to be counted separately, we have a combination problem.

Recap

In this section we have discussed:

- The fundamental counting rule.
- The factorial rule.
- The permutations rule (when items are all different).
- The permutations rule (when some items are identical to others).
- The combinations rule.

Examples

Pgs 183-185: 2, 6, 8, 14, 21, 27, 32