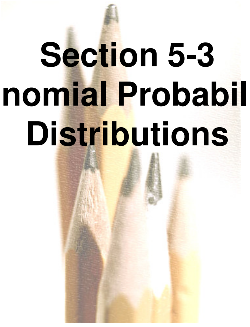


Section 5-3
Binomial Probability Distributions

Binomial Probability Distribution

A **binomial probability distribution** results from a procedure that meets all the following requirements:

1. The procedure has a **fixed number of trials**.
2. The trials must be **independent**. (The outcome of any individual trial doesn't affect the probabilities in the other trials.)
3. Each trial must have all outcomes classified into **two categories** (commonly referred to as **success** and **failure**).
4. The probability of a success remains the same in all trials.

1

Notation for Binomial Probability Distributions

S and F (success and failure) denote the two possible categories of all outcomes; p and q will denote the probabilities of S and F, respectively, so

$P(\text{S}) = p$ (p = probability of success)

$P(\text{F}) = 1 - p = q$ (q = probability of failure)

Notation (continued)

n denotes the fixed number of trials.

x denotes a specific number of successes in *n* trials, so *x* can be any whole number between 0 and *n*, inclusive.

p denotes the probability of success in *one* of the *n* trials.

q denotes the probability of failure in one of the *n* trials.

P(*x*) denotes the probability of getting exactly *x* successes among the *n* trials.

Method 1: Using the Binomial Probability Formula

$$P(x) = \frac{n!}{(n-x)!x!} \cdot p^{x} \cdot q^{n-x}$$

for $x = 0, 1, 2, \ldots, n$

where

n = number of trials

x = number of successes among n trials

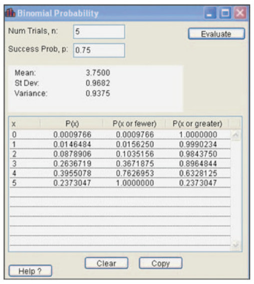
p = probability of success in any one trial

q = probability of failure in any one trial ($q = 1 - p$)

Method 2: Using Technology

STATDISK, Minitab, Excel, SPSS, SAS and the TI-83/84 Plus calculator can be used to find binomial probabilities.

STATDISK



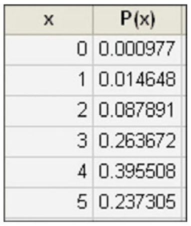
Num Trials, n: 5

Success Prob, p: 0.75

Mean: 3.7500
St Dev: 0.9682
Variance: 0.9375

x	P(x)	P(x or fewer)	P(x or greater)
0	0.0009766	0.0009766	1.0000000
1	0.0146484	0.0156250	0.9990234
2	0.0878906	0.1035156	0.9843750
3	0.2636719	0.3671875	0.8964844
4	0.3955078	0.7626953	0.6328125
5	0.2373047	1.0000000	0.2373047

MINITAB

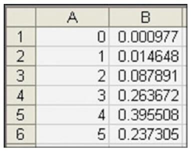


x	P(x)
0	0.000977
1	0.014648
2	0.087891
3	0.263672
4	0.395508
5	0.237305

Method 2: Using Technology

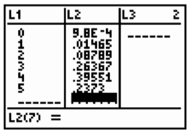
STATDISK, Minitab, Excel and the TI-83 Plus calculator can all be used to find binomial probabilities.

EXCEL



	A	B
1	0	0.000977
2	1	0.014648
3	2	0.087891
4	3	0.263672
5	4	0.395508
6	5	0.237305

TI-83 PLUS Calculator



```
L1      |L2      |L3     2
 0      | 9.8E-4 | ------
 1      | .01465 |
 2      | .08789 |
 3      | .26367 |
 4      | .39551 |
 5      | .2373  |
 ------ |        |
L2(7) =
```

Rationale for the Binomial Probability Formula

$$P(x) = \underbrace{\frac{n\,!}{(n-x\,)!x!}}\cdot\ p^{x}\ \cdot\ q^{n-x}$$

The number of outcomes with exactly *x* successes among *n* trials

Binomial Probability Formula

$$\mathrm{P(x)} = \underbrace{\frac{n\,!}{(n-x\,)!x!}}\cdot\ \underbrace{p^{x}\ \cdot\ q^{n-x}}$$

Number of outcomes with exactly *x* successes among *n* trials

The probability of *x* successes among *n* trials for any one particular order

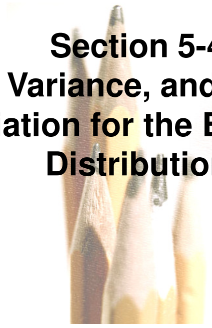
Recap

In this section we have discussed:

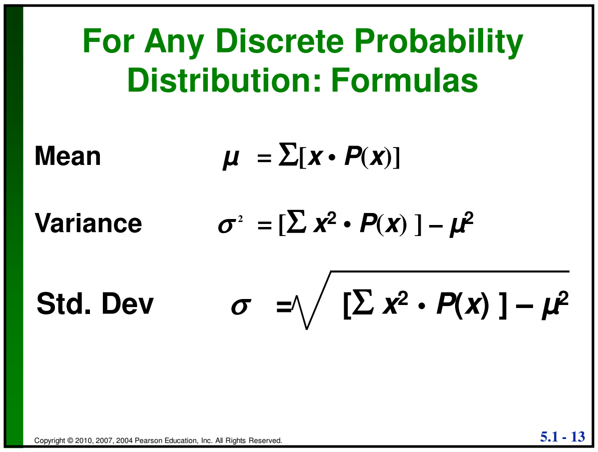
- The definition of the binomial probability distribution.
- Notation.
- Important hints.
- Three computational methods.
- Rationale for the formula.

Examples

EX: 7, 9, 13, 15, 25, 30, 36



Section 5-4 Mean, Variance, and Standard Deviation for the Binomial Distribution

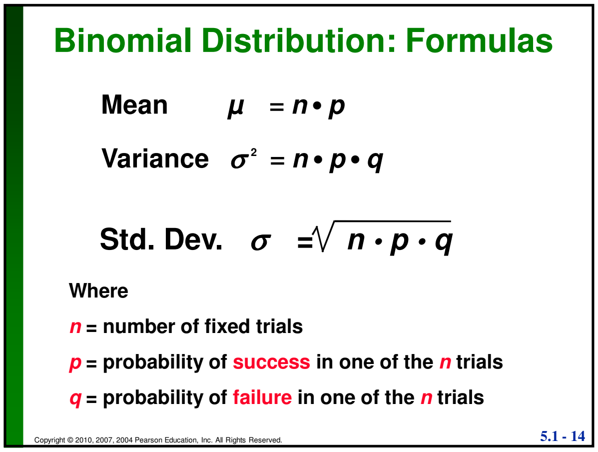


For Any Discrete Probability Distribution: Formulas

Mean $\mu = \Sigma[x \cdot P(x)]$

Variance $\sigma^2 = [\Sigma x^2 \cdot P(x)] - \mu^2$

Std. Dev $\sigma = \sqrt{[\Sigma x^2 \cdot P(x)] - \mu^2}$



Binomial Distribution: Formulas

Mean $\mu = n \cdot p$

Variance $\sigma^2 = n \cdot p \cdot q$

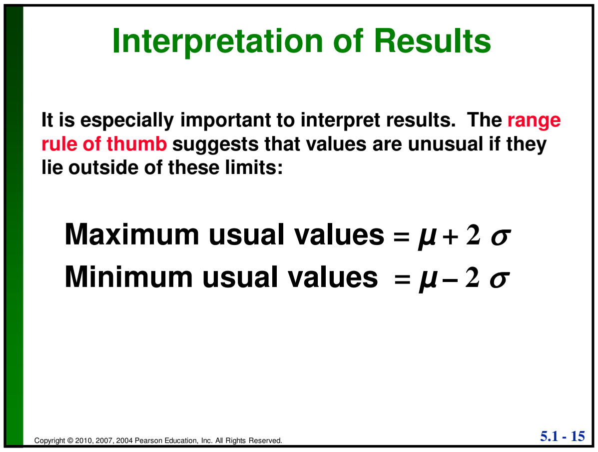
Std. Dev. $\sigma = \sqrt{n \cdot p \cdot q}$

Where

n = number of fixed trials

p = probability of success in one of the n trials

q = probability of failure in one of the n trials



Interpretation of Results

It is especially important to interpret results. The range rule of thumb suggests that values are unusual if they lie outside of these limits:

Maximum usual values $= \mu + 2\,\sigma$

Minimum usual values $= \mu - 2\,\sigma$

Recap

In this section we have discussed:

- **Mean, variance and standard deviation formulas for any discrete probability distribution.**
- **Mean, variance and standard deviation formulas for the binomial probability distribution.**
- **Interpreting results.**

Examples

EX: 5, 9, 13, 17, 20