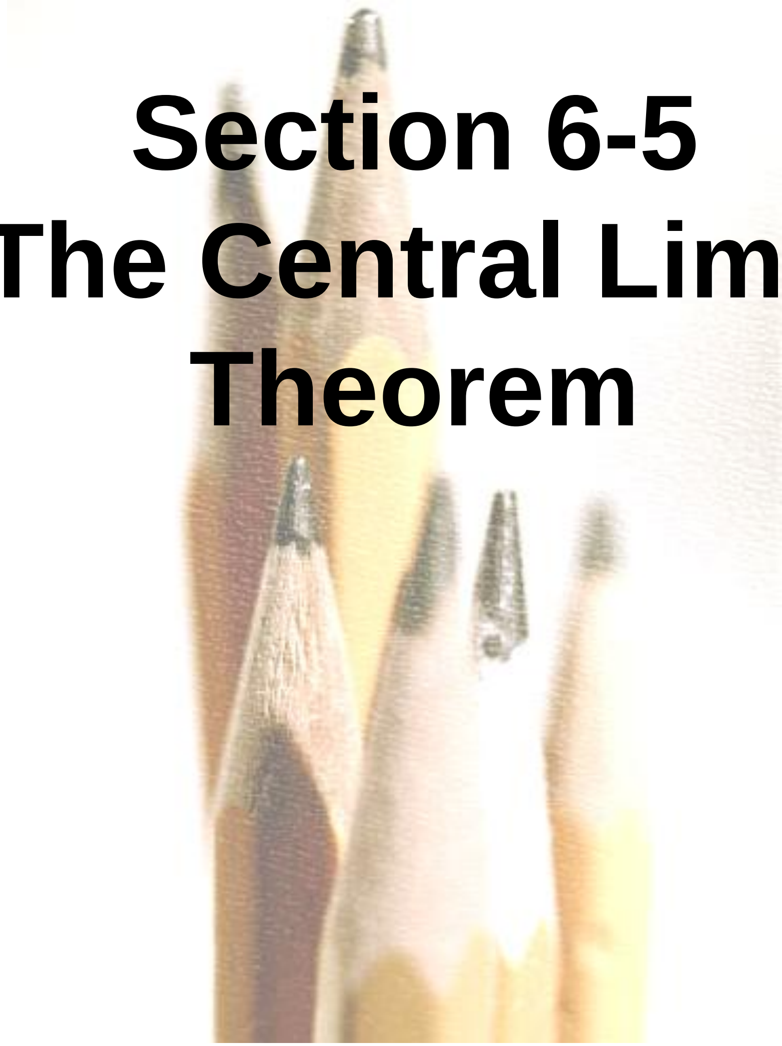
Section 6-5
The Central Limit Theorem



Key Concept

The *Central Limit Theorem* tells us that for a population with *any* distribution, the distribution of the sample means approaches a normal distribution as the sample size increases.

The procedure in this section form the foundation for estimating population parameters and hypothesis testing.

Central Limit Theorem

Given:

1. The random variable *x* has a distribution (which may or may not be normal) with mean μ and standard deviation σ.
2. Simple random samples all of size *n* are selected from the population. (The samples are selected so that all possible samples of the same size *n* have the same chance of being selected.)

Central Limit Theorem – cont.

Conclusions:

1. **The distribution of sample $\bar{x}$ will, as the sample size increases, approach a normal distribution.**

2. **The mean of the sample means is the population mean μ.**

3. **The standard deviation of all sample means is $\sigma/\sqrt{n}$.**

Practical Rules Commonly Used

1. **For samples of size *n* larger than 30, the distribution of the sample means can be approximated reasonably well by a normal distribution. The approximation gets closer to a normal distribution as the sample size *n* becomes larger.**
2. **If the original population is *normally distributed*, then for any sample size *n*, the sample means will be normally distributed (not just the values of *n* larger than 30).**

Notation

the mean of the sample means

$$\mu_{\bar{x}} = \mu$$

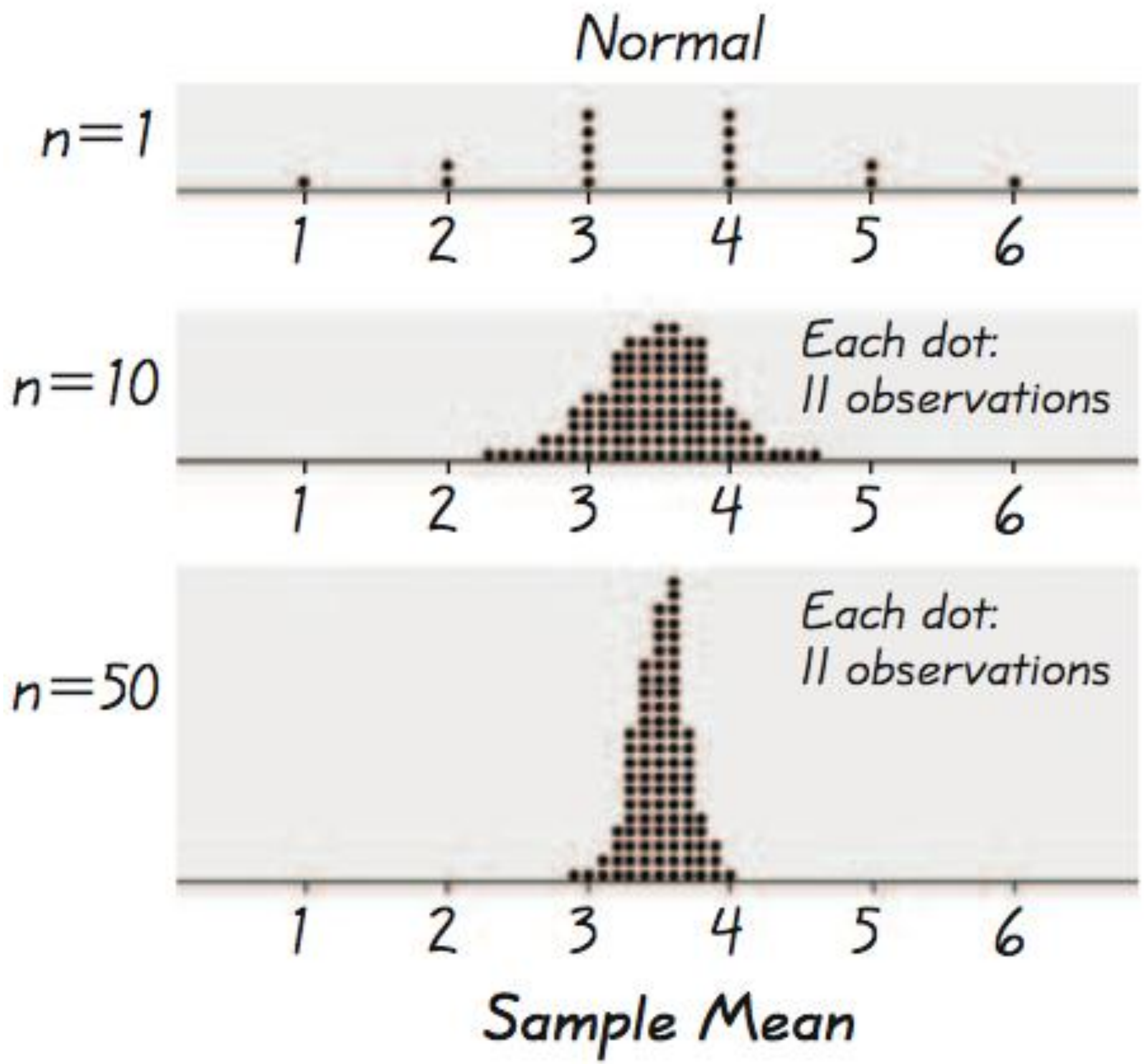
the standard deviation of sample mean

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

(often called the standard error of the mean)

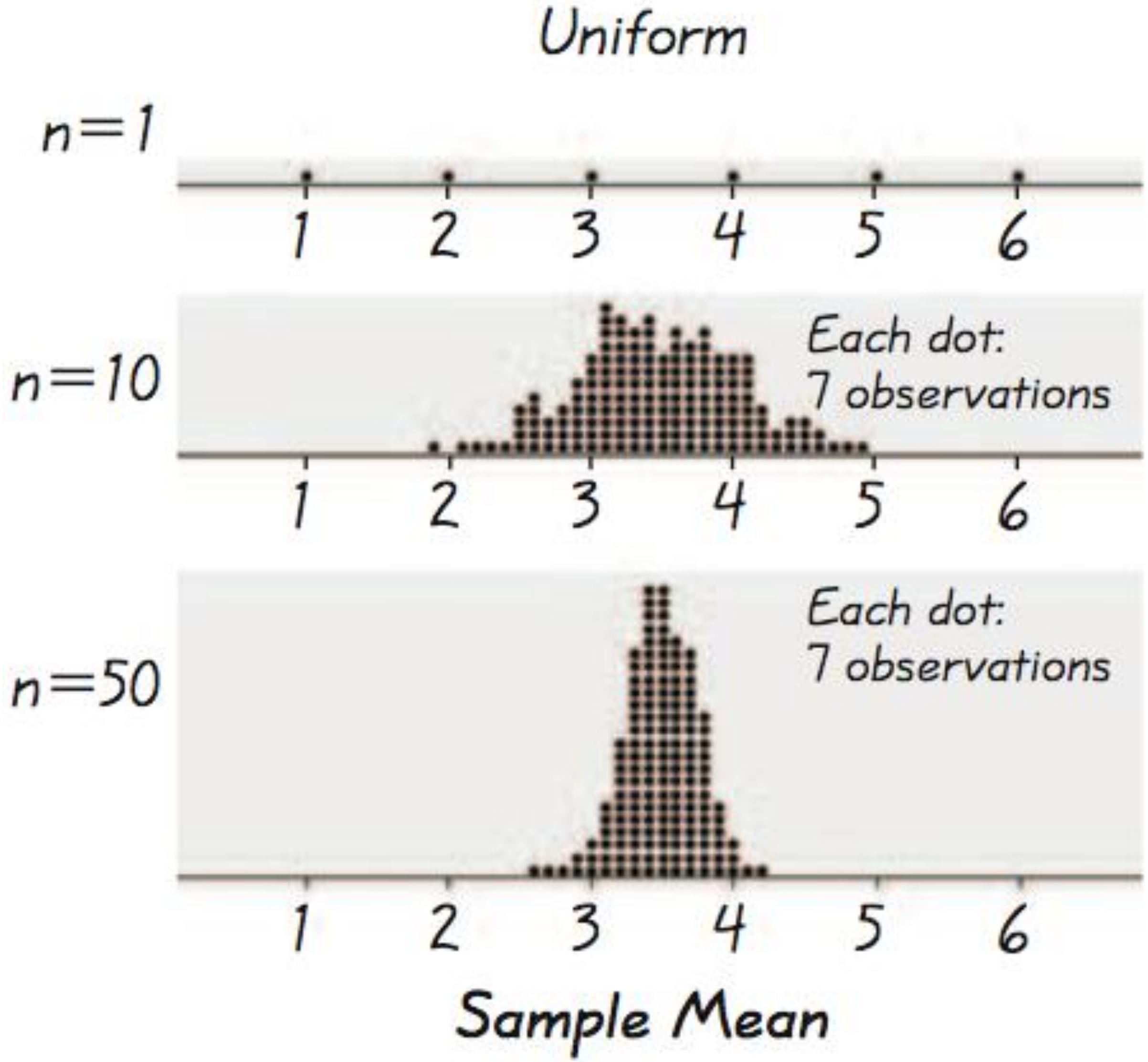
Example - Normal Distribution

As we proceed from $n = 1$ to $n = 50$, we see that the distribution of sample means is approaching the shape of a normal distribution.



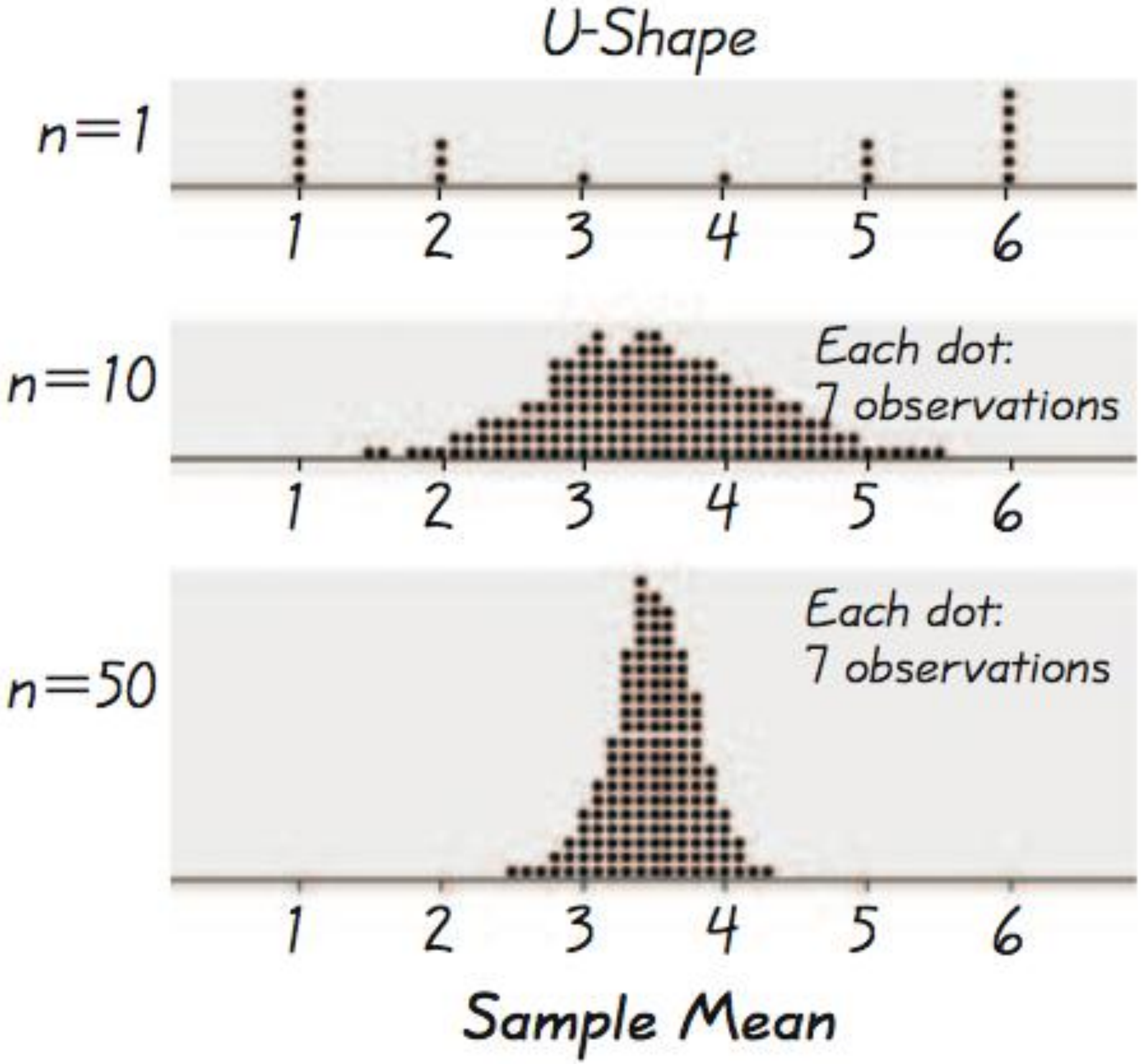
Example - Uniform Distribution

As we proceed from *n* = 1 to *n* = 50, we see that the distribution of sample means is approaching the shape of a normal distribution.



Example - U-Shaped Distribution

As we proceed from *n* = 1 to *n* = 50, we see that the distribution of sample means is approaching the shape of a normal distribution.



Important Point

As the sample size increases, the sampling distribution of sample means approaches a normal distribution.

Example – Water Taxi Safety

Use the Chapter Problem. Assume the population of weights of men is normally distributed with a mean of 172 lb and a standard deviation of 29 lb.

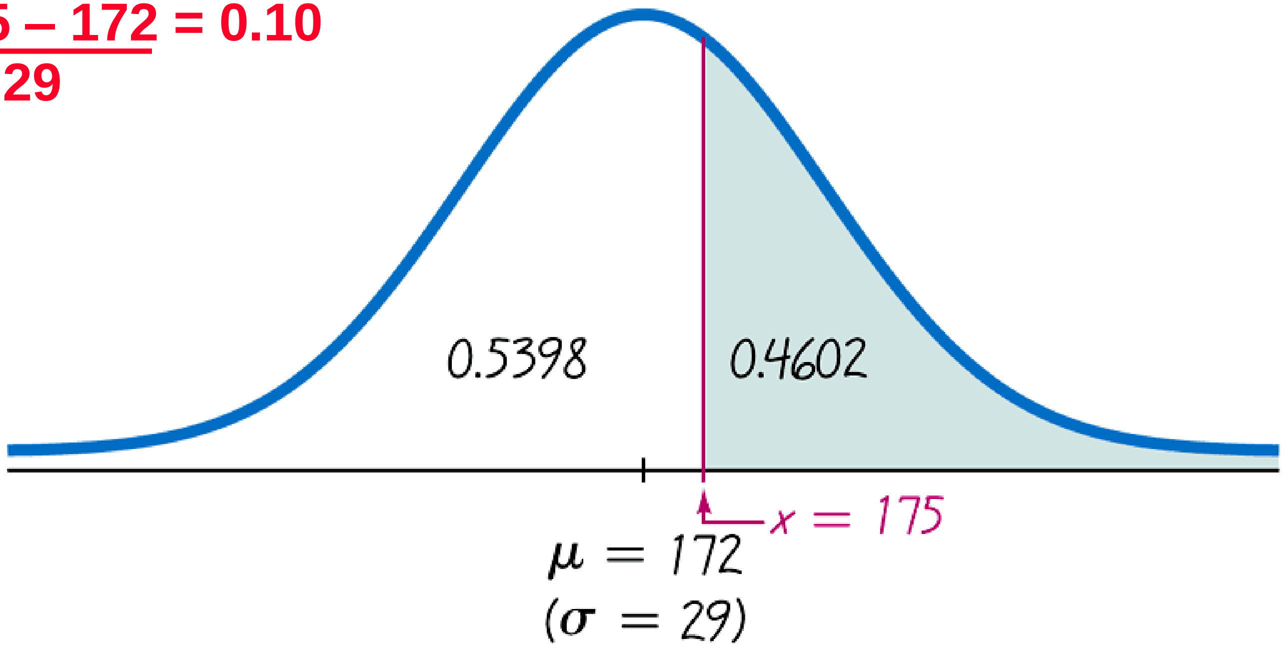
a) Find the probability that if an *individual* man is randomly selected, his weight is greater than 175 lb.

b) b) Find the probability that *20 randomly selected men* will have a mean weight that is greater than 175 lb (so that their total weight exceeds the safe capacity of 3500 pounds).

Example – cont

a) Find the probability that if an *individual* man is randomly selected, his weight is greater than 175 lb.

$$z = \frac{175 - 172}{29} = 0.10$$

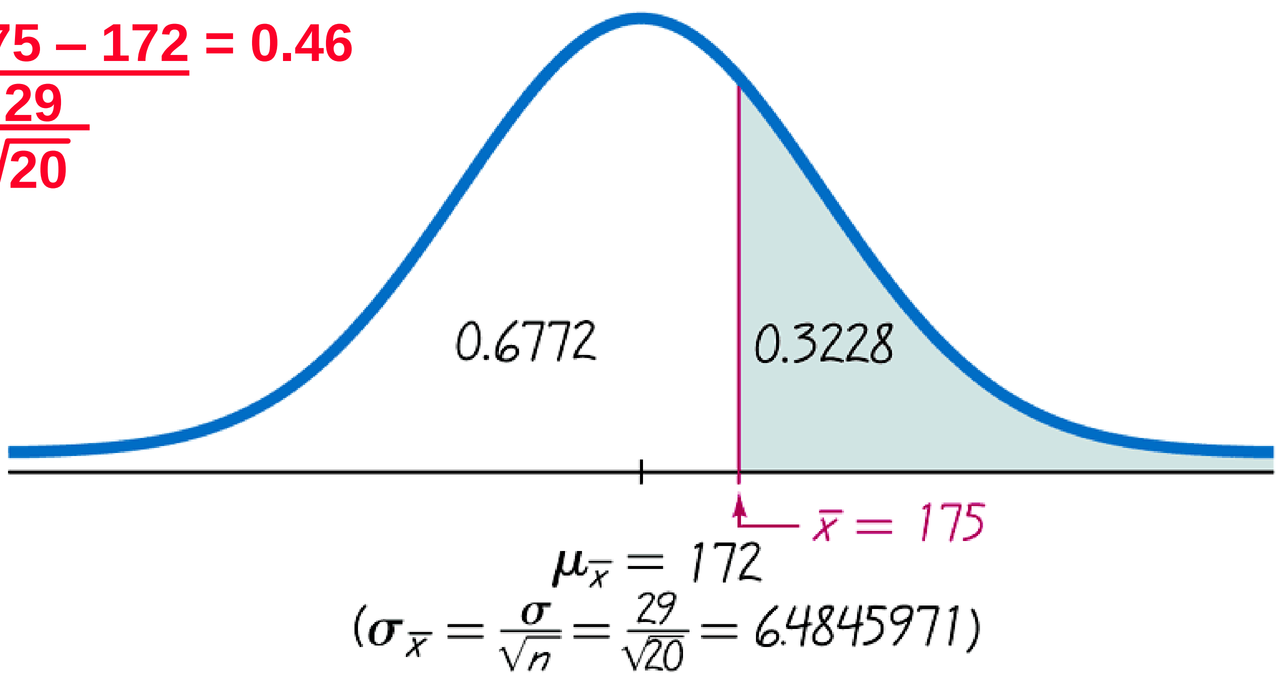


(a)

Example – cont

b) Find the probability that *20 randomly selected men* will have a mean weight that is greater than 175 lb (so that their total weight exceeds the safe capacity of 3500 pounds).

$$z = \frac{175 - 172}{\frac{29}{\sqrt{20}}} = 0.46$$



(b)

Example - cont

a) Find the probability that if an *individual* man is randomly selected, his weight is greater than 175 lb.

$$P(x > 175) = 0.4602$$

b) Find the probability that *20 randomly selected men* will have a mean weight that is greater than 175 lb (so that their total weight exceeds the safe capacity of 3500 pounds).

$$P(\bar{x} > 175) = 0.3228$$

It is much easier for an individual to deviate from the mean than it is for a group of 20 to deviate from the mean.

Interpretation of Results

Given that the safe capacity of the water taxi is 3500 pounds, there is a fairly good chance (with probability 0.3228) that it will be overloaded with 20 randomly selected men.

Correction for a Finite Population

When sampling without replacement and the sample size *n* is greater than 5% of the finite population of size *N* (that is, $n > 0.05N$), adjust the standard deviation of sample means by multiplying it by the *finite population correction factor*:

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \sqrt{\frac{N-n}{N-1}}$$

finite population correction factor

Recap

In this section we have discussed:

- **Central limit theorem.**
- **Practical rules.**
- **Effects of sample sizes.**
- **Correction for a finite population.**

Section 6-6
Normal as Approximation to Binomial

Key Concept

This section presents a method for using a normal distribution as an approximation to the binomial probability distribution.

If the conditions of $np \geq 5$ and $nq \geq 5$ are both satisfied, then probabilities from a binomial probability distribution can be approximated well by using a normal distribution with mean $\mu = np$ and standard deviation $\sigma = \sqrt{npq}$.

Review

Binomial Probability Distribution

1. The procedure must have a **fixed number of trials.**
2. The trials must be **independent.**
3. Each trial must have all outcomes classified into **two categories** (commonly, success and failure).
4. The probability of success remains the same in all trials.

Solve by binomial probability formula, Table A-1, or technology.

Approximation of a Binomial Distribution with a Normal Distribution

$np \geq 5$

$nq \geq 5$

then $\mu = np$ and $\sigma = \sqrt{npq}$

and the random variable has

a  **distribution.**

(normal)

Procedure for Using a Normal Distribution to Approximate a Binomial Distribution

1. Verify that both $np \geq 5$ and $nq \geq 5$. If not, you must use software, a calculator, a table or calculations using the binomial probability formula.

2. Find the values of the parameters μ and σ by calculating $\mu = np$ and $\sigma = \sqrt{npq}$.

3. Identify the discrete whole number *x* that is relevant to the binomial probability problem. Focus on this value temporarily.

Procedure for Using a Normal Distribution to Approximate a Binomial Distribution

4. **Draw a normal distribution centered about μ, then draw a *vertical strip area* centered over *x*. Mark the left side of the strip with the number equal to *x* – 0.5, and mark the right side with the number equal to *x* + 0.5. *Consider the entire area of the entire strip to represent the probability of the discrete whole number itself.***

5. **Determine whether the value of *x* itself is included in the probability. Determine whether you want the probability of at least *x*, at most *x*, more than *x*, fewer than *x*, or exactly *x*. Shade the area to the right or left of the strip; also shade the interior of the strip *if and only if x itself* is to be included. This total shaded region corresponds to the probability being sought.**

Procedure for Using a Normal Distribution to Approximate a Binomial Distribution

6. Using x – 0.5 or x + 0.5 in place of x, find the area of the shaded region: find the z score; use that z score to find the area to the left of the adjusted value of x; use that cumulative area to identify the shaded area corresponding to the desired probability.

Example – Number of Men Among Passengers

Finding the Probability of "At Least 122 Men" Among 213 Passengers

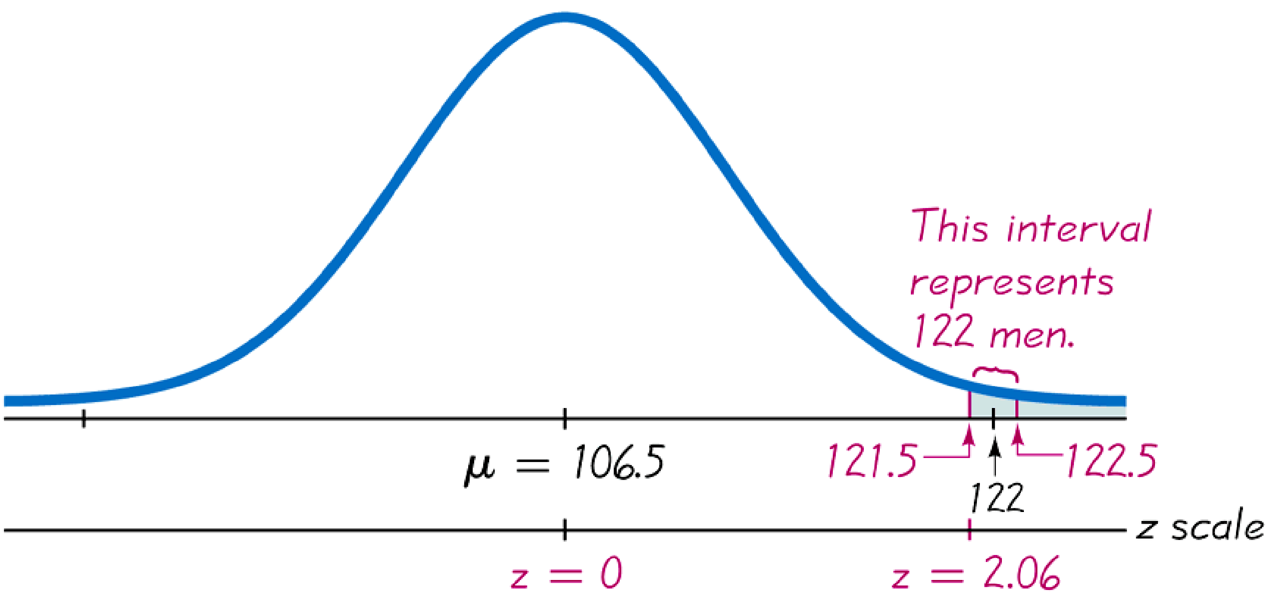


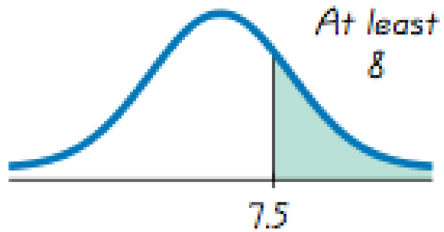
Figure 6-21

Definition

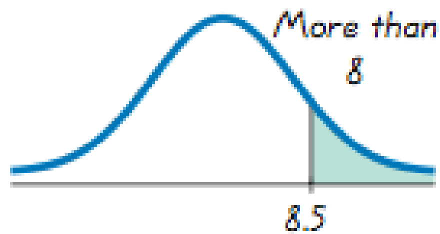
When we use the normal distribution (which is a continuous probability distribution) as an approximation to the binomial distribution (which is discrete), a continuity correction is made to a discrete whole number x in the binomial distribution by representing the discrete whole number x by the interval from

$$x - 0.5 \text{ to } x + 0.5$$

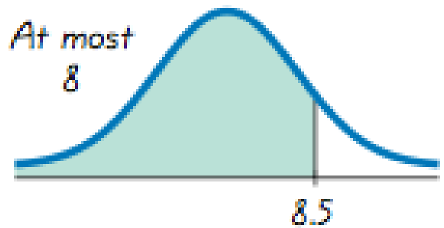
(that is, adding and subtracting 0.5).



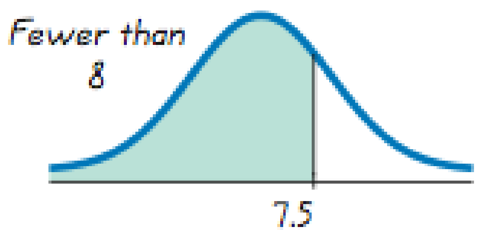
X = **at least** **8**
(includes 8 and above)



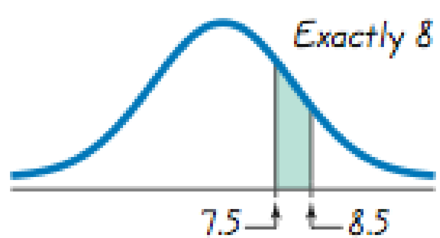
X = **more than** **8**
(doesn’t include 8)



X = **at most** **8**
(includes 8 and below)



X = **fewer than** **8**
(doesn’t include 8)



X = **exactly** **8**

Recap

In this section we have discussed:

- **Approximating a binomial distribution with a normal distribution.**
- **Procedures for using a normal distribution to approximate a binomial distribution.**
- **Continuity corrections.**