Exam 4: Ch 7 & 8
Math 176, Precalculus, Section 6265
Spring 2007: Michael Orr

NAME Answer Key
NO TI-89 CALCULATORS!!!!!!!!

100 points. Show all work to receive full credit. You may use a calculator. CHECK YOUR WORK!!!!

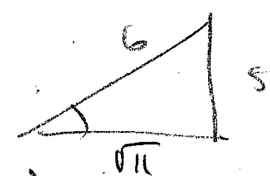
1. (5 pts each) Find the ***exact value*** of each expression. You may need to use a half-angle, double angle, sum or difference formula. You may need to sketch an appropriate right triangle.

a. $\cos 105°$ $= \cos(60° + 45°)$

$= \cos 60 \cos 45 - \sin 60 \sin 45$

$= \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \frac{\sqrt{3}}{2}\left(\frac{\sqrt{2}}{2}\right) = \boxed{\frac{\sqrt{2}-\sqrt{6}}{4}}$

b. Find $\sin 2x$, given $\sin x = -\frac{5}{6}$ with x in quadrant III.



$\cos X = -\frac{\sqrt{11}}{6}$

$\sin 2X = 2 \sin X \cos X = 2\left(-\frac{5}{6}\right)\left(-\frac{\sqrt{11}}{6}\right)$

$= \boxed{\frac{5\sqrt{11}}{18}}$

c. $\sin^{-1}\left(\sin\frac{5\pi}{6}\right)$ $=$ $\sin\frac{5\pi}{6} = \sin\frac{\pi}{6}$

$= \boxed{\frac{\pi}{6}}$

d. $\tan\left(\cos^{-1}\left(\frac{\sqrt{2}}{2}\right)\right)$ $= \boxed{1}$

2. (5 pts) Perform the indicated operation. $(6\sqrt{3} - 6i)(2i)$

$12\sqrt{3}\,i - 12i^2 = \boxed{12 + 12\sqrt{3}\,i}$

3. (4 pts) Express the complex number in trigonometric form: -4 $+ 0i$

$z = r(\cos\theta + i\sin\theta)$

$\boxed{z = 4(\cos\pi + i\sin\pi)}$

4. (4 pts) Convert to rectangular coordinates: $\left(-12, \frac{\pi}{4}\right)$

$x = r\cos\theta$
$= -12\cos\frac{\pi}{4}$
$= -12\frac{\sqrt{2}}{2} = -6\sqrt{2}$

$y = r\sin\theta$
$= -12\sin\frac{\pi}{4}$
$= -6\sqrt{2}$

$\boxed{(-6\sqrt{2}, -6\sqrt{2})}$

5. (6 pts) Given $z_1 = 1+\sqrt{3}\,i$ and $z_2 = 5-3i$, find z_1z_2 and $\frac{z_1}{z_2}$.

$z_1z_2 = (1+\sqrt{3}i)(5-3i)$
$= 5 - 3i + 5\sqrt{3}i - 3\sqrt{3}(i^2)$
$= \boxed{(5+3\sqrt{3}) + (-3+5\sqrt{3})i}$

$10.20 + 5.66i$

$2\sqrt{34}\left[\cos\left(\pi/3 + \tan^{-1}(-3/5)\right) + i\sin\left(\frac{\pi}{3} + \tan^{-1}(-3/5)\right)\right]$

$\frac{z_1}{z_2} = \frac{1+\sqrt{3}i}{(5-3i)}\frac{(5+3i)}{(5+3i)}$
$= \frac{5+3i+5\sqrt{3}i+3\sqrt{3}i^2}{25+9}$
$= \boxed{\frac{(5-3\sqrt{3}) + (3+5\sqrt{3})i}{34}}$

$-0.01 + 0.34i$

$\frac{z_1}{z_2} = \frac{2}{\sqrt{34}}\left[\cos\left(\frac{\pi}{3} - \tan^{-1}\left(\frac{-3}{5}\right)\right) + i\sin\left(\frac{\pi}{3} - \tan^{-1}\left(\frac{-3}{5}\right)\right)\right]$

6. *(2 pts each)* True or False. Determine whether each statement is True or False

a. $2\sin^2 x\cot x = \sin 2x$ is an identity. TRUE

b. $\left\langle \frac{3}{\sqrt{45}}, -\frac{6}{\sqrt{45}} \right\rangle$ is a unit vector. TRUE

c. $\tan^{-1}x$ has domain $[-1,1]$ and range $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. FALSE

ALL REALS

d. $5(\cos\pi + i\sin\pi)$ is the trigonometric form of the number -5. TRUE

e. $\langle 1,-4\rangle$ and $\langle 4,-1\rangle$ are orthogonal vectors. FALSE

7. (6 pts each) Verify that each equation is an identity.

a. $\csc x - \tan\frac{x}{2} = \cot x$

$$\frac{1}{\sin x} - \left(\frac{1-\cos x}{\sin x}\right) = \frac{\cos x}{\sin x} = \cot x$$ ✓

b. $(1-\tan x)(1-\cot x) = 2 - \sec x \csc x$

$$\left(1-\frac{\sin x}{\cos x}\right)\left(1-\frac{\cos x}{\sin x}\right) = \left(\frac{\cos x - \sin x}{\cos x}\right)\left(\frac{\sin x - \cos x}{\sin x}\right)$$

$$= \frac{\cos x \sin x - \cos^2 x - \sin^2 x + \cos x \sin x}{\cos x \sin x}$$

$$= \frac{2\cos x \sin x - (\cos^2 x + \sin^2 x)}{\cos x \sin x}$$

$$= \frac{2\cancel{\cos x \sin x}}{\cancel{\cos x \sin x}} - \frac{1}{\cos x \sin x}$$

$$= \boxed{2 - \sec x \csc x}$$

8. (6 pts) Find all solutions of the equation: $2\tan x \cos x - \sqrt{2}\tan x = 0$

$\tan X(2\cos X - \sqrt{2}) = 0$

$\tan X = 0$ or $2\cos X - \sqrt{2} = 0$

$= \frac{\sin X}{\cos X}$

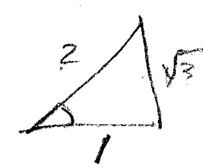
$\cos X = \frac{\sqrt{2}}{2}$

$\boxed{X = 0 \pm K\pi}$

$X = \frac{\pi}{4}, \frac{7\pi}{4}$

$$\boxed{X = 0 \pm K\pi, \ \frac{\pi}{4} + 2K\pi, \ \frac{7\pi}{4} + 2K\pi}$$

9. (6 pts) Find all solutions of the equation in the interval $[0, 2\pi)$: $\sin 2x = -\frac{\sqrt{3}}{2}$



Q III & Q IV

$2X = \frac{4\pi}{3}, \frac{5\pi}{3}$

$X = \frac{4\pi}{6}, \frac{5\pi}{6}$

$\boxed{X = \frac{2\pi}{3}, \frac{5\pi}{6}}$

$\frac{2X}{2} = \frac{4\pi}{3 \cdot 2} + \frac{2K\pi}{2}$

or

$\frac{2X}{2} = \frac{5\pi}{3 \cdot 2} + \frac{2K\pi}{2}$

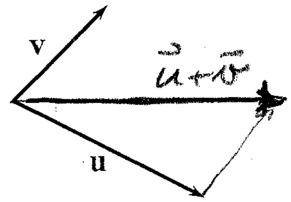
$\boxed{\frac{2\pi}{3}, \frac{5\pi}{6}, \frac{5\pi}{3}, \frac{11\pi}{6}}$

$X = \frac{4\pi}{6} + K\pi = \frac{2\pi}{3} + K\pi$

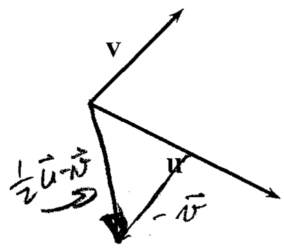
$\frac{5\pi}{6} + K\pi = \frac{5\pi}{6} + K\pi$

For the vectors **u** and **v** graphed,

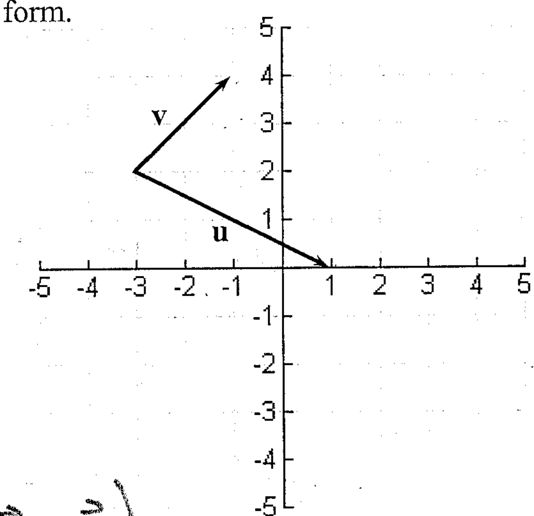
6. (2 pts) Sketch the vector $\mathbf{u}+\mathbf{v}$



7. (2 pts) Sketch the vector $\frac{1}{2}\mathbf{u}-\mathbf{v}$.



8. (2 pts each) Using the grid, write **u** and **v** in component form.



u = $\langle 4, -2\rangle = 4\vec{i} - 2\vec{j}$

v = $\langle 2, 2\rangle = 2\vec{i} + 2\vec{j}$

9. (2 pts) Find the magnitude of **u**, denoted $|\mathbf{u}|$

$|u| = \sqrt{4^2+2^2} = \sqrt{20}$

$\boxed{|u| = 2\sqrt{5}}$

10. (2 pts) Find $4\mathbf{u}-\frac{1}{2}\mathbf{v}$

$4(4\vec{i}-2\vec{j}) - (\vec{i}+\vec{j})$

$16\vec{i} - 8\vec{j} - \vec{i} - \vec{j} =$

$\boxed{15\vec{i} - 9\vec{j}}$

$\boxed{\langle 15, -9\rangle}$

11. (2 pts) Find $\mathbf{u}\cdot\mathbf{v}$ $= 8 + (-4) = \boxed{4}$

12. (2 pts) Find the angle θ between **u** and **v**. Round to the nearest degree.

$\cos\theta = \frac{u\cdot v}{|u||v|} = \frac{4}{(2\sqrt{5})(2\sqrt{2})} = \frac{4}{4\sqrt{10}} = \frac{1}{\sqrt{10}}$

$\theta = 71.57^\circ \quad \doteq \boxed{72^\circ}$

14. (5 pts) Write $4-4i$ in trigonometric form.

$r = \sqrt{4^2+4^2} = 4\sqrt{2}$

$\tan\theta = \frac{-4}{4} = -1 \Rightarrow \theta = \cancel{\frac{3\pi}{4}}, \frac{7\pi}{4}$

$4-4i = \boxed{4\sqrt{2}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)}$

15. (6 pts) Find $(-3+3i)^5$ using DeMoivre's Theorem. You must show all work.

$r = 3\sqrt{2}$ QII

$\tan\theta = \frac{3}{-3} = -1 \Rightarrow \theta = \frac{3\pi}{4}$

$z = 3\sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$

$z^5 = (3\sqrt{2})^5\left(\cos\frac{15\pi}{4} + i\sin\frac{15\pi}{4}\right)$

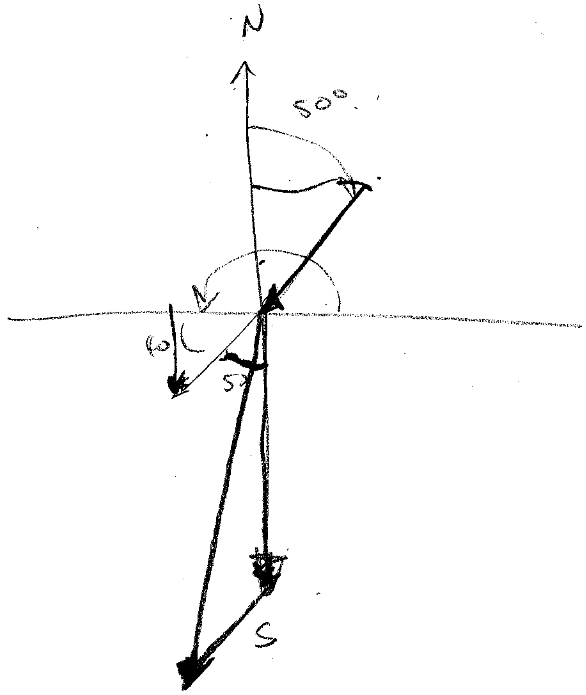
$= (243)(4\sqrt{2})\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)$

$= 972\sqrt{2}\left(\frac{\sqrt{2}}{2} + i\left(-\frac{\sqrt{2}}{2}\right)\right)$

$\boxed{z^5 = 972 - 972i}$

***BONUS** (total of **10** extra points)*

A plane is flying due south with an airspeed of 294 mph. A wind from a direction of 50° is blowing at 9 mph. Find the bearing and speed of the plane. (HINT: Resolve the wind blowing from 50° into a heading relative to S.)



PLANE $= -294\vec{j}$

WIND $= 9\cos(220)\hat{i} + 9\sin(220)\hat{j}$

$= -6.8944\hat{i} - 5.7851\vec{j}$

RESULTANT $= -6.8944\hat{i} - 299.7851\vec{j}$

MAGNITUDE $= \sqrt{(-6.8944)^2 + (-299.7851)^2}$

$= 299.86$ MPH

$\approx$ 300 MPH

BEARING: $\tan\theta = \dfrac{-299.7851}{-6.8944}$

$= 88.68°$

$\Rightarrow$ 1.32° W of S

$\boxed{\text{S } 1.32° \text{ W @ 300 MPH}}$