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Math 178 Practice Exam #2 **Chapter 9** **100 points possible**

Name: Solution Set

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1) A computer dealer can sell 1500 personal computers per week at a price of \$3000 each. He estimates that each \$200 price reduction will result in 300 more sales per week.

Begin by finding the following;

p(x) = $3000 - 200x$

q(x) = $1500 + 300x$

R(x) = $p(x) \cdot q(x) = (3000 - 200x)(1500 + 300x)$

$= 4500000 - 300000x + 900000x - 60000x^2$

$\boxed{R(x) = 4{,}500{,}000 + 600{,}000x - 60{,}000x^2}$

What price should he charge to maximize his revenue?

$R'(x) = 600{,}000 - 120{,}000x = 0$

$x = 5 \qquad p(x) = 3000 - 200(5) = \boxed{\$2000}$

How many computers will he sell at that price?

$q(x) = 1500 + 300(5) = \boxed{3000 \text{ computers}}$

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2) Find the absolute extreme values of the function
$f(x) = x^4 - 4x^3 + 4x^2$ on $[0,3]$

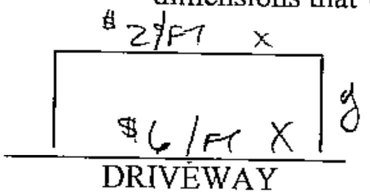
$$f'(x) = 4x^3 - 12x^2 + 8x = 0$$
$$4x(x^2 - 3x + 2) = 0$$
$$4x(x-2)(x-1) = 0$$
$$CV: \; x = 0, 1, 2$$

$$f''(x) = 12x^2 - 24x + 8$$
$$= 4(3x^2 - 6x + 2)$$

$$f(0) = 0$$
$$f(1) = 1$$
$$f(2) = 0$$
$$f(3) = 9$$

ABSOLUTE EXTREME VALUES
MIN: $(0,0)$ & $(2,0)$
MAX: $(3,9)$

3) Sara wants to build a garden surrounded by a fence along her driveway. If the garden is to be 800 square feet, and the fence along the driveway costs \$6 per foot whereas on the other three sides it costs only \$2 per foot, find the dimensions that will minimize the cost. Also find the minimum cost.



$$A = 800\text{ FT}^2 = xy \Rightarrow y = \frac{800}{x}$$
$$C(x) = \$2(x + 2y) + 6x$$
$$= 2x + 4y + 6x$$
$$= 8x + 4y$$

$$C(x) = 8x + 4\left(\frac{800}{x}\right)$$
$$C(x) = 8x + 3200x^{-1}$$
$$C'(x) = 0 = 8 - 3200x^{-2}$$
$$8 = \frac{3200}{x^2} \qquad x^2 = \frac{3200}{8}$$
$$x^2 = 400$$
$$x = 20\text{ FT}$$
$$y = \frac{800}{20} = 40\text{ FT}$$

20 FT $\perp$ TO DRIVEWAY
40 FT $\parallel$ TO DRIVEWAY
COST = \$320

4) Suppose that the relationship between the tax rate t on imported shoes and the total sales S (in millions of dollars) is given by $s(t) = 8 - 15\sqrt[3]{t}$ FIND THE TAX RATE "t" that maximizes revenue for the government.

$S(t) = 8 - 15\, t^{1/3}$

$R(t) = t\,S(t) = 8t - 15t^{4/3}$

$R'(t) = 8 - \frac{4}{\cancel{3}}(\overset{5}{\cancel{15}}\, t^{1/3}) = 0$

$\frac{8}{20} = \frac{20\, t^{1/3}}{20}$

$t^{1/3} = 0.4$

$(t^{1/3})^3 = (0.4)^3 \Rightarrow t = 0.064 = \boxed{6.4\%}$

5) An automobile dealer expects to sell 400 cars a year. The cars cost \$11,000 each plus a fixed charge of \$500 per delivery. If it costs \$1,000 to store a car for a year, find the order size and the number of orders that minimize inventory costs.

Ave # of cars $= \frac{x}{2}$

COST PER ORDER $= 11{,}000x + 500$

SIZE of ORDER $= \frac{400}{x}$

STORAGE COST $= \$1000 \left(\frac{x}{2}\right) = 500x$

ORDER COSTS = COST/ORDER × # of ORDERS

TOTAL = STORAGE COST + ORDER COSTS

$= 500x + (11{,}000x + 500)\left(\frac{400}{x}\right)$

$= 500x + 4{,}400{,}000 + \frac{200000}{x} = 500x + 4{,}400{,}000 + 200{,}000x^{-1}$

$T' = 0 = 500 - 200{,}000x^{-2}$

$500 = \frac{200{,}000}{x^2}$ $\quad x^2 = 400$

$x = 20$

ORDER SIZE IS 20 CARS PER ORDER FOR TOTAL OF 20 ORDERS FOR THE YEAR

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6) Use implicit differentiation to find $\frac{dy}{dx}$ if $5 = -3x + y^4 - 2x^2y^3$

$$0 = -3 + 4y^3\frac{dy}{dx} - 2\left[2xy^3 + x^2(3y^2)\frac{dy}{dx}\right]$$

$$= -3 + 4y^3\frac{dy}{dx} - 4xy^3 - 6x^2y^2\frac{dy}{dx}$$

$$\frac{3+4xy^3}{4y^3-6x^2y^2} = \frac{(4y^3-6x^2y^2)\frac{dy}{dx}}{4y^3-6x^2y^2}$$

$$\boxed{\frac{dy}{dx} = \frac{3+4xy^3}{4y^3-6x^2y^2}}$$

7) A company's demand equation is $x = \sqrt{68-p^2}$ where p is the price in dollars.

Find $\frac{dp}{dx}$ when p =2 . **Interpret your answer.**

$$x = (68-p^2)^{1/2}$$

$$1 = \frac{1}{2}(68-p^2)^{-1/2}(-2p)\frac{dp}{dx}$$

$$1 = \frac{-p}{\sqrt{68-p^2}}\frac{dp}{dx} \Rightarrow \frac{dp}{dx} = \frac{\sqrt{68-p^2}}{-p}$$

$$\left.\frac{dp}{dx}\right|_{p=2} = \frac{\sqrt{68-2^2}}{-2} = \frac{\sqrt{64}}{-2} = \boxed{-4}$$

EACH \$4 DECREASE IN PRICE WILL INCREASE DEMAND BY 1 UNIT

8) The number of traffic accidents per year in a city of population "P" is predicted to be $T = 0.002p^{3/2}$. If the population is growing by a **rate of 500 people a year**, find the rate at which traffic accidents will be rising when the population is P = 40,000 .

$$T = 0.002p^{3/2} \qquad \frac{dp}{dt} = 500 \ \& \ p = 40{,}000$$

$$T' = \frac{3}{2}(0.002p^{1/2})\frac{dp}{dt}$$

$$T' = \frac{3}{2}(0.002\sqrt{40000})(500)$$

$$T' = 300$$

RATE AT WHICH TRAFFIC ACCIDENTS IS INCREASING IS 300 ACCIDENTS / YEAR