Formula Sheet

Kinematics & Forces

$v_x = \frac{dx}{dt}$ $a_x = \frac{dv_x}{dt}$

$\bar{v}_x = \frac{\Delta x}{\Delta t}$ $\bar{a}_x = \frac{\Delta v_x}{\Delta t}$

Constant acceleration

$x(t) = x_o + v_o t + ½ at^2$

$v(t) = v_o + at$

$v_f^2 = v_o^2 + 2a\Delta x$

$\bar{v} = (v_f + v_0)/2$

$x(t) = x_o + vt - ½ at^2$

(compare the above eqn. carefully to the first constant acc. eqn. before using.)

Projectile motion

$v_x(t) = v_{ox}$

$x(t) = x_o + v_{ox}t$

$v_y(t) = v_{oy} - gt$

$y(t) = y_o + v_{oy}t - ½ gt^2$

$\Sigma \mathbf{F} = m\mathbf{a}$

Uniform circular motion

$a_c = \frac{v^2}{r}$

$v = \frac{2\pi r}{T}$ $T = \frac{1}{f}$

Friction

$f_s \leq \mu_s F_n$

$f_k = \mu_k F_n$

$F_{grav} = mg$

$F_{spring} = -kx$

Vector Operations

$\vec{A} \bullet \vec{B} = AB\cos\theta$

$\vec{A} \bullet \vec{B} = A_x B_x + A_y B_y + A_z B_z$

$\vec{A} \times \vec{B} = AB\sin\theta\hat{n}$

i × j = k

j x k = i

k x i = j

Energy & Momentum

$K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$

$W = \int F_{||}\, dx \; (= \int \mathbf{F}\bullet d\mathbf{r})$

$W_{net} = \Delta E_{SYSTEM}$

$= \Delta E_{Mech} + \Delta E_{Th}$

$\Delta E_{Th} = f_k \Delta s$

$P_{instantaneous} = \frac{dW}{dt} = \mathbf{F}\bullet\mathbf{v}$

$\Delta U = -W$ of conservative force

$F(x) = -\frac{dU(x)}{dx}$

$U_{spring}(x) = \frac{1}{2}kx^2$

$U_g(y) = mgy$ [near the earth's surface]

$\mathbf{p} = m\mathbf{v}$

I = Impulse $= \int \mathbf{F}\, dt = \mathbf{p}_f - \mathbf{p}_i$

$= \mathbf{F}_{avg} \Delta t = \Delta \mathbf{p}$

Collisions:

$\mathbf{P}_{tot,f} = \mathbf{P}_{tot,i}$

Elastic collisions:

$K_{tot,f} = K_{tot,i}$

Center of mass:

$M_{tot}\, \mathbf{r}_{cm} = m_1\mathbf{r_1} + m_2\mathbf{r_2} + ...$

$M_{tot}\, \mathbf{v}_{cm} = m_1\mathbf{v_1} + m_2\mathbf{v_2} + ...$

$M_{tot}\, \mathbf{a}_{cm} = m_1\mathbf{a_1} + m_2\mathbf{a_2} + ...$

$\mathbf{P_{tot}} = \mathbf{p_{cm}}$

$= M_{tot}\, \mathbf{v}_{cm} = m_1\mathbf{v_1} + m_2\mathbf{v_2} + ...$

$\Sigma \mathbf{F_{ext}} = \frac{d\mathbf{P}_{tot}}{dt}$

Rotation

$\Delta\theta = \Delta s/r$

$\omega = \frac{d\theta}{dt}$ $\alpha = \frac{dw}{dt}$

$v_{tang} = \omega r$

$a_{tang} = \alpha r$

$a_{rad} = \frac{v^2}{r} = \omega^2 r$

Constant acceleration

$\theta(t) = \theta_o + \omega_o t + \frac{1}{2}\alpha t^2$

$\omega(t) = \omega_o + \alpha t$

$\omega_f^2 = \omega_o^2 + 2\alpha\Delta\theta$

$\bar{\omega} = (\omega_f + \omega_0)/2$

$I = \Sigma m_i r_i^2 \; (= \int r^2 dm)$

$I_{cm} = mR^2$, ring

$I_{cm} = ½mR^2$, disk

$I_{cm} = \frac{2}{5}mR^2$, solid sphere

$I_{cm} = mL^2/12$ for rod

$I_{end} = mL^2/3$ for rod

$I_d = I_{cm} + Md^2$

(parallel axis theorem.)

$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$

$|\tau| = rF\sin\theta = r_\perp F = r F_\perp$

$\Sigma\tau = I\,\boldsymbol{\alpha}$

$W = \int \tau\, d\theta$

$K = \frac{1}{2}I\omega^2$

Rolling without slipping:

$v_{cm} = \omega R$

$a_{cm} = \alpha R$

Angular momentum

$\mathbf{L} = \mathbf{r} \times \mathbf{p}$

$|\mathbf{L}| = rp\sin\theta = r_\perp p = r p_\perp$

$L = I\omega$

$\Sigma\tau_{ext} = \frac{d\mathbf{L}_{tot}}{dt}$

Gravitation

$$|F_g| = \frac{GMm}{r^2}$$

The force on mass 1 due to mass 2. The unit vector points to mass 2.

$$\vec{F}_{12} = \frac{GMm}{r^2}\hat{r}$$

Principle of Superposition

$$\vec{F}_{1,net} = \sum_{i=2}^{n} \vec{F}_{1i}$$

Gravitational P.E.

$$U_g(r) = -\frac{GMm}{r}$$

Oscillations in simple harmonic motion.

$x(t) = x_{max}\cos(\omega t + \phi)$
so $d^2x/dt^2 = -\omega^2 x(t)$

$\theta(t) = \theta_m \cos(\omega t + \phi)$
so $d^2\theta/dt^2 = -\omega^2\theta(t)$

$$\omega = 2\pi f = \frac{2\pi}{T}$$

$$\omega = \sqrt{\frac{k}{m}} \text{ : spring}$$

$$T = 2\pi\sqrt{\frac{I}{MgD}} \text{ : physical pendulum}$$

Fluids

$p_h = p_o + \rho g h$

$g = 9.8\ m/s^2$
$G = 6.67 \times 10^{-11}\ Nm^2/kg^2$
$R_{earth} = 6.38 \times 10^6\ m$
$M_{earth} = 5.97 \times 10^{24}\ kg$

Units related to Pressure

$1\ Pa = 1\ N/m^2$
$1\ atm = 1.013 \times 10^5\ Pa$
$\rho_{water} = 1000\ kg/m^3$

Law of Cosines: $C^2 = A^2 + B^2 - 2AB\cos\gamma$

Law of Sines: $$\frac{\sin\alpha}{A} = \frac{\sin\beta}{B} = \frac{\sin\gamma}{C}$$