Formula Sheet: Electricity and Magnetism

Coulomb's law

$$\vec{F}_{1,2} = \frac{kq_1q_2}{r_{1,2}^2}\hat{r}_{1,2}$$

The force on charge 1 due to charge 2. The unit vector points from charge 2 to charge 1

Electric Fields

$$\vec{F}(\text{on } q_0) = q_0\vec{E}$$

Field of a point charge $\vec{E} = \frac{kq}{r^2}\hat{r}$

Principle of superposition $\vec{E}_{net} = \sum_{i=1}^{n}\vec{E}_i$

Field from an infinitesimal charge element. $d\vec{E} = \frac{kdq}{r^2}\hat{r}$

Field on either side of an infinite charged plane $|E| = \frac{\sigma}{2\varepsilon_0}$

Discontinuity at the surface of charged plane $|\Delta E| = \frac{\sigma}{\varepsilon_0}$

Gauss's law

Flux: defined: $\phi_{net} = \oint_S \vec{E}\cdot\hat{n}dA = \oint_S E_n dA$

Gauss's Law: $\phi_{net} = 4\pi k q_{enclosed}$

$$= \frac{q_{enclosed}}{\varepsilon_0}$$

Electric potential

$$V(r) = \frac{kq}{r}$$

The potential for a point charge or outside a spherically symmetric charge distribution, with V=0 at infinity.

$dV = \frac{kdq}{r}$ potential from an infinitesimal charge element.

Potential calculated from the electric field

$$dV = -\vec{E}\cdot d\vec{\ell} \text{ and } -\frac{dV}{d\ell} = E_{tan}$$

$$\Delta V = V_b - V_{ref} = -\int_{ref}^{b}\vec{E}\cdot d\vec{\ell}$$

Constants

$e = 1.602 \times 10^{-19}$ C

$1eV = 1.602 \times 10^{-19}$ J

$m_e = 9.11 \times 10^{-31}$ kg

$m_p = 1.67 \times 10^{-27}$ kg

$k = 1/(4\pi\varepsilon_o) = 8.99 \times 10^9$ N m^2/C^2

$\varepsilon_o = 8.85 \times 10^{-12}$ F/m (or C^2/N m^2)

$\mu_o = 4\pi \times 10^{-7}$ T m/A

1 T = 10^4 G

g = 9.8 m/s^2

Differential area element: dA

dA = (circumference)x(thickness)

ring: $dA = 2\pi r\, dr$

Differential volume elements: dv

dv = (surface area)x(thickness)

thin sheet: dv = A dy

thin cylindrical shell: $dv = 2\pi rL\, dr$

thin spherical shell: $dv = 4\pi r^2\, dr$

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Potential Energy

$\Delta U = q_0 \Delta V$ for q_0 moving through ΔV

$U = \sum_{i=1}^{n} q_i V_i$; for a group of point charges bringing in each charge sequentially

$U = \frac{1}{2} QV$ for a conductor at potential V

Energy stored in a capacitor:

$$U = \tfrac{1}{2} CV^2 = \tfrac{1}{2} QV = \tfrac{1}{2} \frac{Q^2}{C}$$

Energy density of electric fields

$$u_e = \tfrac{1}{2} \varepsilon_0 E^2$$

Capacitors

$$C = \frac{Q}{V} \qquad \text{parallel plate}: C = \frac{\varepsilon_0 A}{d}$$

Capacitors in a circuit

parallel: $C_{\text{equiv}} = C_1 + C_2 + \ldots = \sum_{i=1}^{n} C_i$

series:

$$C_{\text{equiv}} = \frac{1}{1/C_1 + 1/C_2 + \ldots} = \frac{1}{\sum_{i=1}^{n} \frac{1}{C_i}}$$

DC Circuits

$$I = \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt} = qnAv_d$$

$$R = \rho \frac{L}{A}$$

Resistors in a circuit

series: $R_{\text{equiv}} = R_1 + R_2 + \ldots = \sum_{i=1}^{n} R_i$

parallel:

$$R_{\text{equiv}} = \frac{1}{1/R_1 + 1/R_2 + \ldots} = \frac{1}{\sum_{i=1}^{n} \frac{1}{R_i}}$$

Ohm's Law

$V = IR$

Power dissipated in a resistor

$$P = IV = I^2 R = \frac{V^2}{R}$$

Kirchhoff's rules

(1) at a junction: $\Sigma I_{in} = \Sigma I_{out}$

(2) around a closed loop: $\Sigma \Delta V = 0$

Time dependent circuits: charge on the *capacitor ($\mathcal{E}$ is the battery emf):*

$$\text{RC, charging}: Q(t) = C\mathcal{E}\left(1 - e^{-t/\tau_{RC}}\right), \; \tau_{RC} = RC$$

$$\text{RC, discharging}: Q(t) = Q_o e^{-t/\tau_{RC}}$$

You can derive I(t) and V(t) from these.

Magnetic Force

$$\vec{F}_B = q\vec{v} \times \vec{B} \quad [\, \vec{F}_{Lorentz} = q(\vec{E} + \vec{v} \times \vec{B}) \,]$$

$$d\vec{F}_B = Id\vec{l} \times \vec{B} \qquad \vec{F}_B = I\vec{L} \times \vec{B}$$

Magnetic Torques on current loops

Magnetic moment: $\vec{\mu} = NIA\hat{n}$

$$\tau = NIAB\sin(\theta) \qquad \vec{\tau} = \vec{\mu} \times \vec{B}$$

potential energy of a current loop

$$U = -\vec{\mu} \cdot \vec{B}$$

Kinematics & Forces

$v_x = \frac{dx}{dt}$ $a_x = \frac{dv_x}{dt}$

$\bar{v}_x = \frac{\Delta x}{\Delta t}$ $\bar{a}_x = \frac{\Delta v_x}{\Delta t}$

Constant acceleration
(These equations can be used for any coordinate)

$x(t) = x_o + v_o t + ½ at^2$
$v(t) = v_o + at$
$v_f^2 = v_o^2 + 2a\Delta x$
$\bar{v} = (v_f + v_0)/2$
$x(t) = x_o + vt – ½ at^2$

(compare the above eqn. carefully to the first constant acc. eqn. before using.)

Projectile motion (special case of constant acceleration)

$v_x(t) = v_{ox}$
$x(t) = x_o + v_{ox}t$
$v_y(t) = v_{oy} + a_y t$
$y(t) = y_o + v_{oy}t + ½a_y t^2$
(remember that *v* and *a* can be positive or negative)

$\Sigma \mathbf{F} = m\mathbf{a}$

Uniform circular motion

$a_c = \frac{v^2}{r}$

$v = \frac{2\pi r}{T}$ $T = \frac{1}{f}$

Friction

$f_s \le \mu_s F_n$
$f_k = \mu_k F_n$

$F_{grav} = mg$
$F_{spring} = –kx$

Vector Operations

$\vec{A} \bullet \vec{B} = AB\cos\theta$

$\vec{A} \bullet \vec{B} = A_x B_x + A_y B_y + A_z B_z$

$\vec{A} \times \vec{B} = AB\sin\theta\hat{n}$

i × j = k
j x k = i
k x i = j

Energy & Momentum

$K = \frac{1}{2} mv^2 = \frac{p^2}{2m}$

$W = \int F_{\|} dx$ $(= \int \mathbf{F} \cdot d\mathbf{r})$

For mechanical energy and friction

$W_{net} = \Delta E_{SYSTEM}$
$= \Delta E_{Mech} + \Delta E_{Th}$

$\Delta E_{Th} = f_k \Delta s$

For electric fields

$W_{external} = \Delta E_{SYSTEM}$
$= \Delta K + \Delta U$

where ΔU is as given on the previous page.

$P_{instantaneous} = \frac{dW}{dt} = \mathbf{F} \cdot \mathbf{v}$

$\Delta U = –W$ of conservative force

$F(x) = -\frac{dU(x)}{dx}$

$U_{spring}(x) = \frac{1}{2} kx^2$

$U_g(y) = mgy$ [near the earth's surface]

$\mathbf{p} = m\mathbf{v}$

$\mathbf{I} =$ **Impulse** $= \int \mathbf{F}\, dt = \mathbf{p}_f - \mathbf{p}_i$

$= \mathbf{F}_{avg} \Delta t = \Delta \mathbf{p}$

Collisions:
$\mathbf{P}_{tot,f} = \mathbf{P}_{tot,i}$

Elastic collisions:
$K_{tot,f} = K_{tot,i}$

Center of mass:
$M_{tot}\, \mathbf{r}_{cm} = m_1\mathbf{r_1} + m_2\mathbf{r_2} + ...$
$M_{tot}\, \mathbf{v}_{cm} = m_1\mathbf{v_1} + m_2\mathbf{v_2} + ...$
$M_{tot}\, \mathbf{a}_{cm} = m_1\mathbf{a_1} + m_2\mathbf{a_2} + ...$

$\mathbf{P_{tot}} = \mathbf{p_{cm}}$
$= M_{tot}\, \mathbf{v}_{cm} = m_1\mathbf{v_1} + m_2\mathbf{v_2} + ...$

$\Sigma \mathbf{F_{ext}} = \frac{d\mathbf{P}_{tot}}{dt}$

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Rotation

$\Delta\theta = \Delta s/r$

$\omega = \frac{d\theta}{dt} \quad \alpha = \frac{dw}{dt}$

$v_{tang} = \omega r$

$a_{tang} = \alpha r$

$a_{rad} = \frac{v^2}{r} = \omega^2 r$

Constant acceleration

$\theta(t) = \theta_o + \omega_o t + \frac{1}{2}\alpha t^2$

$\omega(t) = \omega_o + \alpha t$

$\omega_f^2 = \omega_o^2 + 2\alpha\Delta\theta$

$\overline{\omega} = (\omega_f + \omega_0)/2$

$I = \Sigma m_i r_i^2 \ (= \int r^2 dm)$

$I_{cm} = mR^2$, ring

$I_{cm} = ½mR^2$, disk

$I_{cm} = \frac{2}{5}mR^2$, solid sphere

$I_{cm} = mL^2/12$ for rod

$I_{end} = mL^2/3$ for rod

$I_d = I_{cm} + Md^2$

(parallel axis theorem.)

$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$

$|\boldsymbol{\tau}| = rF\sin\theta = r_\perp F = r F_\perp$

$\Sigma\boldsymbol{\tau} = I\boldsymbol{\alpha}$

$W = \int \tau \, d\theta$

$K = \frac{1}{2} I\omega^2$

Rolling without slipping:

$v_{cm} = \omega R$

$a_{cm} = \alpha R$

Angular momentum

$\mathbf{L} = \mathbf{r} \times \mathbf{p}$

$|\mathbf{L}| = rp\sin\theta = r_\perp p = r p_\perp$

$L = I\omega$

$\Sigma\boldsymbol{\tau}_{ext} = \frac{d\mathbf{L}_{tot}}{dt}$