The secret of success in Chapter 15

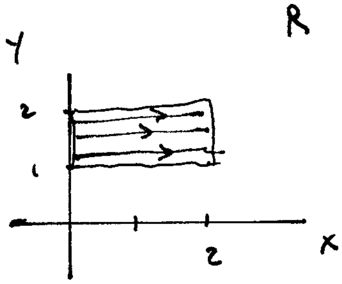
You only need three skills:

1. Be able to calculate iterated integrals.
2. Be able to set up an iterated integral which will represent the double integral (geometry of $R \to$ limits of the iterated integral)
3. Be able to look at the limits on an iterated integral and understand what the region, R, of integration is (limits $\to$ geometry of R).

15.2 (cont'd) Iterated integrals over rectangular regions

16) Use Fubini's Theorem to evaluate

$$\iint_R (y + xy^{-2})\, dA \quad \text{where } R = \{(x,y) \mid 0 \le x \le 2,\ 1 \le y \le 2\}$$



(1) $$\int_1^2 \int_0^2 (y + xy^{-2})\, dx\, dy \qquad \text{OR}$$

(2) $$\int_0^2 \int_1^2 (y + xy^{-2})\, dy\, dx$$

Use (1): $$\int_1^2 \left[xy + \tfrac{1}{2}x^2 y^{-2}\right]_0^2 dy = \int_1^2 \left[(2y + 2y^{-2}) - 0\right] dy$$

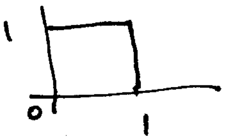
$$= \int_1^2 (2y + 2y^{-2})\, dy = \left[y^2 - 2y^{-1}\right]_1^2$$

$$= \left(2^2 - 2\cdot\tfrac{1}{2}\right) - \left(1^2 - 2\cdot\tfrac{1}{1}\right) = 3 - (-1) = 4$$

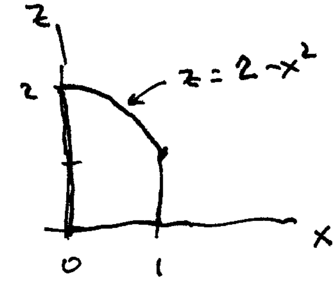
Sketch the <u>solid</u> whose volume is this integral:

24) $$\int_0^1 \int_0^1 (2 - x^2 - y^2)\, dy\, dx$$ [This is Skill #3.]

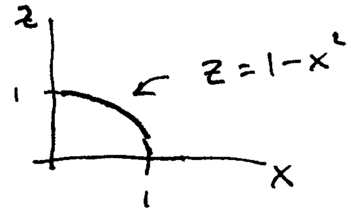
Top view:

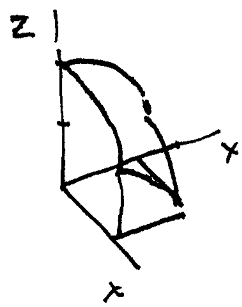


Side views: In the plane $y=0$ (xz-plane)

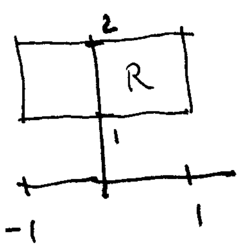


In the plane $y=1$





28) [Set up only] Find the volume of the solid <u>under</u> $z = 3y^2 - x^2 + 2$ and <u>above</u> $R = [-1, 1] \times [1, 2]$



$$\int_{-1}^{1} \int_{1}^{2} (3y^2 - x^2 + 2)\, dy\, dx = \text{(etc)}$$

<u>Remark</u>: If R is a rectangle (whose sides are parallel to the coordinate axes) and $f(x,y) = g(x)\,h(y)$, then the iterated integral can be written as the <u>product</u> single integrals.

<u>ex</u>:
$$\int_0^3 \int_0^{\pi/2} 3x^2 \cos y \, dy\, dx = \int_0^3 3x^2 \left[\int_0^{\pi/2} \cos y \, dy\right] dx$$

This will be a <u>constant</u>.

$$= \left[\int_0^{\pi/2} \cos y\, dy\right] \cdot \left[\int_0^3 3x^2\, dx\right]$$

$$= \left[\sin y\right]_0^{\pi/2} \quad \left[x^3\right]_0^3$$

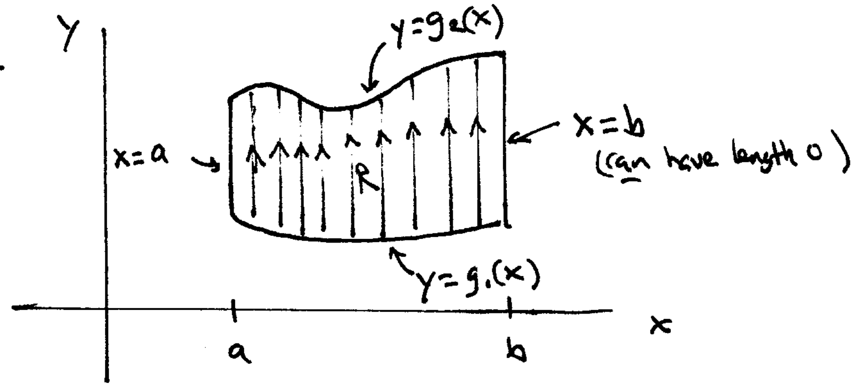
$$= \left(\sin \tfrac{\pi}{2} - \sin 0\right)\left(3^3 - 0^3\right) = (1)(27) = 27$$

15.3 Double integrals over General Regions

Add these numbers:

			10	10
		16	8	24
	24	12	6	42
32	16	8	4	60
32	40	36	28	136

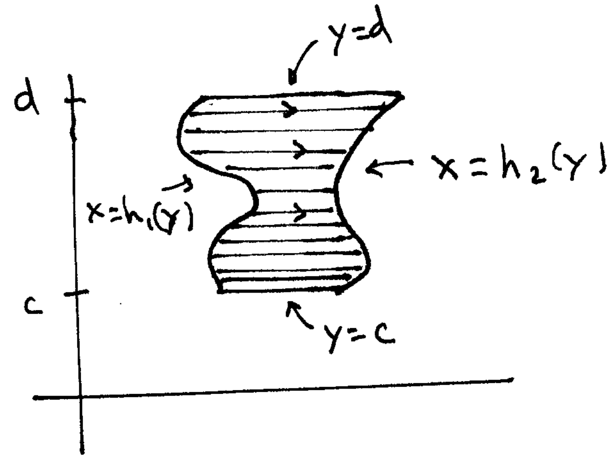
<u>Type I region</u>



The double integral $\iint_R f(x,y)\,dA$ can be calculated as

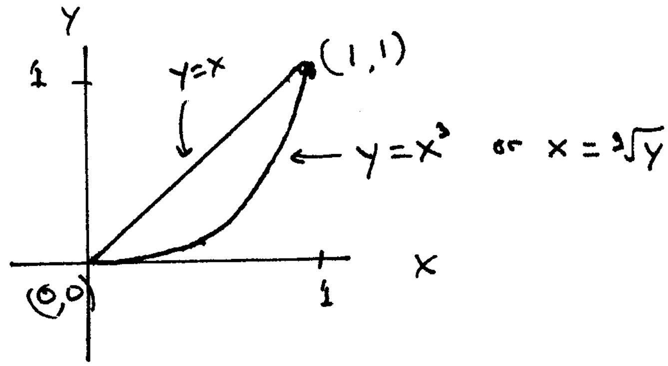
the iterated integral $\displaystyle\int_{x=a}^{x=b}\int_{y=g_1(x)}^{y=g_2(x)} f(x,y)\,dy\,dx$

<u>Type II region</u>



$$\iint_R f(x,y)\,dA = \int_c^d \int_{h_1(y)}^{h_2(y)} f(x,y)\,dx\,dy$$

15.3 18) $\iint_D (x^2+2y)\,dA$ D is bounded by $y=x$, $y=x^3$, $x \geq 0$.



As a type I region  $\int_0^1 \int_{x^3}^{x} (x^2+2y)\,dy\,dx$

$$= \int_0^1 \left[x^2 y + y^2\right]_{x^3}^{x} dx$$

$$= \int_0^1 \left[(x^2 \cdot x + x^2) - (x^2 \cdot x^3 + (x^3)^2)\right] dx$$

$$= \int_0^1 \left[(x^3 + x^2) - (x^5 + x^6)\right] dx$$

$$= \int_0^1 (-x^6 - x^5 + x^3 + x^2)\,dx$$

$$= \left[-\frac{x^7}{7} - \frac{x^6}{6} + \frac{x^4}{4} + \frac{x^3}{3}\right]_0^1$$

$$= -\frac{1}{7} - \frac{1}{6} + \frac{1}{4} + \frac{1}{3} = -\frac{12}{84} - \frac{14}{84} + \frac{21}{84} + \frac{28}{84}$$

$$= \frac{23}{28}$$

As a type II region, this integral could have also been set up as

$$\int_0^1 \int_y^{\sqrt[3]{y}} (x^2+2y)\,dx\,dy$$